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SOME EFFECTS OF
A LAYERED SYSTEM ON DILATIONAL WAVES

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Contract AF 19(604)-7399
Project 8652, Task 865201

TECHNICAL REPORT 3

July 31, 1964

Prepared for

Air Force Cambridge Research Laboratories

Office of Aerospace Research

United States Air Force

Bedford, Massachusetts
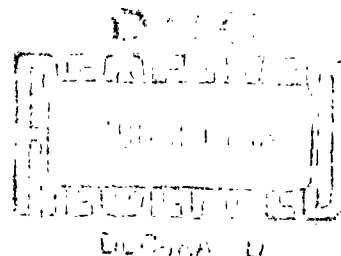
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WORK SPONSORED BY ADVANCED RESEARCH PROJECTS AGENCY

PROJECT VELA-UNIFORM

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Abstract

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Abstract

In this report, the effect of the crustal model on the variation of the surface motion with the angle of incidence and the frequency is examined for several crustal models. The study was carried out by programming the problem for the IBM 1620 and 7072 computer systems using the matrix formulation originally suggested by Thomson and perfected by Haskell and Dorman. From these programs, the ratios of the displacements at the free surface to the total amplitude at depth were computed for several crustal models in ranges of frequency and angle of incidence of interest in seismology. These ratios are, in effect, transmission coefficients.

Six crustal models having such features as thin low-velocity surface layers, low-velocity layers at depth, and relatively thick and relatively thin total thicknesses were considered. For each model, the transmission coefficients were computed for frequencies ranging from .02 cps to 10.0 cps in steps of .02 cps, and for angles of incidence ranging from 21 degrees to 53 degrees in steps of 4 degrees. Haskell's model was included in these calculations in order to obtain a check on the calculations. A further check was obtained by using the transmission coefficients to synthesize the surface motion due to an incident wave of the form $(1/\pi)$ $(\sin 20\pi t)/t$ and comparing these values with

those predicted by ray theory. The agreement was very good.

As a result of these calculations, the importance of the frequency dependent character of the crustal effect has been further emphasized. It has been shown that a thin low-velocity surface layer causes a large variation in the transmission coefficients, while a low-velocity layer at depth has little effect, especially at low frequencies. Further, it has been shown that the total crustal thickness is one of the most significant factors in determining the variation with frequency. The transmission coefficients of relatively thick crustal models have a much more rapid frequency variation than those of the relatively thin crust. At frequencies less than .2 cps, it is very difficult to distinguish between crustal models of very nearly the same thickness by means of the frequency variation of the transmission coefficients unless the internal structure is very different.

SOME EFFECTS OF A LAYERED SYSTEM ON DILATATIONAL WAVES

1. Introduction

With the advent of large networks of calibrated seismographs, it has become increasingly important to determine the effect of the layering in the earth's crust beneath the individual stations on the surface motion generated by seismic waves striking the base of the crust. This report studies some aspects of this problem by calculating the particle motion which would be observed at the surface of a system of plane, horizontal, non-attenuating layers as a result of a continuous sinusoidal plane wave striking the base of the layered system at oblique angles of incidence. (Fig. 1) The Haskell-Thomson matrix method is used to obtain the steady-state response, and then the transient response for a simple pulse shape is obtained by Fourier integration.

Although the relatively recent matrix formulation is used in this report, the observations and basic theory which indicated that the crustal layering could have a significant effect on the surface motion have long been known. In 1929, Imamura observed small oscillations after the first arrival. He suggested these oscillations could be due to the layering in the earth's crust. Then in 1932 Suzuki observed a variation in the apparent angle of incidence with frequency. Since these phenomena could not be explained on the basis of a homogeneous half space, various Japanese investigators began theoretical studies to determine the effect of layering in the crust on body waves.

(0) ———————————— FREE SURFACE ————————————→ + x

1

(1) ——

2

(2) ——

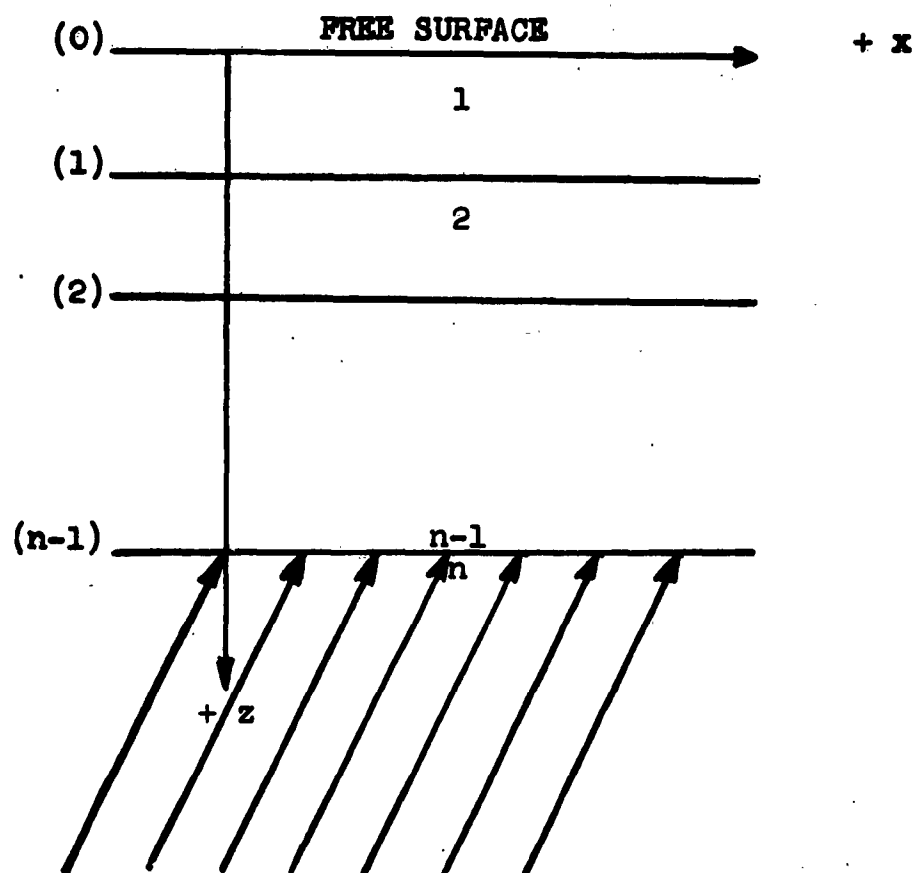(n-1) ————————————————— n-1 ———————————————
n

+ z

FIGURE 1. THE DIRECTION OF THE AXES AND THE LABELLING
OF THE LAYERS IN THE N - LAYER MODEL

Under the conditions of continuity of stress and displacement at the layer boundaries, such investigators as Sezawa (1930), Hasegawa (1930), Sezawa and Kanai (1932a, b, c, 1934; 1935, 1936) and Nishimura and Kanai (1933) showed that layering in the crust could cause the effects mentioned above. Although in principle this fact was well established by 1936, in practice little was known quantitatively about the magnitude of the effect of the layering on the surface motion. Those calculations which were carried out usually involved such simplifying assumptions as normal incidence, equality of density, reduction of the problem to two layers (i.e. one layer over a half-space), and a value of Poisson's ratio equal to 0.25. However, from these studies, especially those of Sezawa and Kanai (1934, 1935), it was possible to state that the effect of the layering varied with both frequency and angle of incidence.

In 1939 Nishimura and Takayama (1939a, 1939b) considered the effect of a single two-layer system for nine angles of incidence with thirty values of steady-state frequency at each angle. This work represented the most extensive set of calculations in existence until 1962, when N. A. Haskell published his work. In this interval the observations of Gutenberg (1957) and Nuttli and Whitmore (1961) gave even more emphasis to the importance of the crustal effect and its variation with frequency, as did the model studies of Press, Oliver and Ewing (1954), Ivanova (1959) and Lavergne (1961). In addition, investigators such as Lindsay (1939), Kanai (1953, 1957), Kanai and Yoshizawa (1959) carried on other theoretical studies. However, the theory and method of approach remained basically the same, and the primarily qualitative conclusions may be summarized as follows:

1) The small oscillations after the arrival of a body phase can
be caused by the crustal structure rather than the source
motion.

2) The effect of the crust varies with

   a) crustal structure

   b) angle of incidence

   c) frequency.

3) Elliptical particle motions from dilatational sources can be
explained on the basis of layering of the crust.

4) Low-velocity sedimentary surface layers can cause large ampli-
tudes to be observed at the free surface.

5) The effect of the crust is nearly constant for waves with very
long wavelength.

These conclusions point out the need to consider the effect of the crust
on any seismic study which is based on amplitudes or on the associated
apparent angle of incidence, especially since the observations are often
carried out on instruments with different frequency response curves.

Some seismologists have attempted to use the frequency dependence
of the surface motion on a layered crust as a means of determining
crustal structure. In particular, several investigators have attempted
to use the spectrum of the ground motion to determine the crustal layer-
ing (Clewell and Simon, 1950, Berzon, 1956a, b; Berzon and Epinatyeva,
1950; and Khudzinsky, 1961). However, because of the lack of comparable
quantitative theoretical studies, their work is mainly of interest as
one of the first attempts to apply the frequency variation of the

surface motion as a tool to determine crustal structure.

In addition to the lack of quantitative theoretical studies, the above work suffered from its inability to determine whether the frequency variation of the spectrum of the ground motion arises from the crustal interference or from the source motion.

As will be seen in section 2, the effect of the source can be eliminated, or at least minimized, by considering the ratio of the horizontal surface motion to the vertical surface motion as a function of frequency. Some attempts to follow this line of invesgation were carried out both theoretically and experimentally by Malinovskaya (1959), Ivanova (1960); Halperin and Folova (1960); and Halperin (1962). Although this method appeared to be superior to the use of the spectrum of a single component, these investigations again suffered from a lack of quantitative theoretical values which could be used for comparison.

Two events combined to provide a means of obtaining theoretical quantitative values on a sufficient scale to be able to separate the effects of the model, the angle of incidence, and the frequency. These were: a) the development of the matrix formulation of the boundary conditions, and b) the general availability of high speed computers.

The present state of the matrix formulation of the boundary conditions is primarily due to three investigators: Thomson (1950); Haskell (1953); and Dorman (1962). Thomson made the original formulation in which he treated the problem of propagation of plane waves through a system of plane parallel layers. Haskell (1953) corrected an error in Thomson's work and emphasized the application of the method to the dispersion of Love

and Rayleigh waves. In 1962 Dorman showed that the previous formulations had, in effect, over-specified the boundary conditions at a boundary between a liquid and a solid. Although the applications to surface waves received the major attention of seismologists until 1960 and 1962 when Haskell considered the applications to body waves, the original formulation was quite general. As early as 1953, Matumoto considered the transmission of body waves using matrix techniques. At that time, even with this powerful tool he was not able to consider as many cases as had Nishimura and Takayama (1937, 1939a). However, the matrix techniques are well suited to modern digital computers and with their coming it has become possible to treat systems of many layers for many angles of incidence and many steady-state frequencies.

In 1962 Haskell obtained contour plots of the amplitude and phase to the vertical and horizontal surface motions for a two-layer system for angles of incidence ranging from zero to 90 degrees and for frequencies ranging from .01 to .333 cps. In 1963 Phinney and Smith obtained theoretical ratios of the spectra in the low frequency range for a number of crustal models. In 1964 Phinney showed a series of curves illustrating the results of a large number of calculations of the theoretical ratios of the vertical surface motion to the horizontal surface motion as a function of frequency.

These calculations are undoubtedly the most extensive set of calculations which have been done in the frequency range from 0.001 to 0.2 cps. From these calculations he concluded:

1) The effects of intermediate and deep crustal structure
   is isolated in the low frequency range.

2) The positions of peaks in the ratio do not change appre-
   ciably with changes in the angle of incidence.

3) Fine structure in the transfer ratio is due to the structure
   of the upper mantle.

4) Thin, low velocity surface layers have no singular effects
   on the crustal peaks in the low frequency range.

He compared these computations with the observations of the ratios of
the spectra of the ground motion at Albuquerque, New Mexico, and Bermuda-
Columbia. From his analysis he was able to obtain crustal-upper mantle
models which were geologically satisfying. Thus, in addition to obtain-
ing considerable information about the theoretical variation of the
spectral ratios, he was able to obtain and use observational studies
for the determination of crustal structure.

2. Theory

Although the matrix formulation of the boundary conditions in a layered system is of considerable importance at the present time, it will not be developed in detail here because of the many excellent treatments which already exist (Haskell, 1953; Dorman, 1963). However, since the application of this formulation to body waves has recently received more attention, we will briefly consider the theoretical background of this application.

As a result of his analysis in 1953, N. A. Haskell obtained the following equation governing the steady-state transmission of plane sinusoidal waves through layered systems:

$$
\begin{vmatrix}
\Delta_n' + \Delta_n'' \\
\Delta_n' - \Delta_n'' \\
\omega_n' - \omega_n'' \\
\omega_n' + \omega_n''
\end{vmatrix}
= E_n^{-1} \, A_{n-1} \, A_{n-2} \;\; \cdots \;\; A_1
\begin{vmatrix}
\dot{u}_o/c \\
\dot{w}_o/c \\
\sigma_o \\
T_o
\end{vmatrix}
\tag{1}
$$

where Δ and ω are plane wave solutions to the dilatational and rotational wave equations. The singly primed terms are the amplitudes of the downward travelling components while the doubly primed terms correspond to the upward travelling waves.

E_n^{-1} is a 4×4 matrix associated with the boundary condition between the half-space and the $n\text{-}1^{th}$ layer. This matrix is independent of frequency.

A_n-1 is a 4×4 matrix associated with transmission across the n-1th layer and the boundary between the n-1th layer and the n-2nd layer. Matrices of this type depend on the layer constants, the angle of incidence and the frequency of the input plane wave.

$\overset{\bullet}{u}_o$ is the horizontal component of the particle velocity at the zeroth interface (Fig. 1)

$\overset{\bullet}{w}_o$ is the vertical component of the particle velocity at the zeroth interface.

c is the apparent surface velocity.

σ_o is the normal stress at the zeroth interface.

$\widehat{T}_o$ is the tangential stress at the zeroth interface.

α_n is the dilatational wave velocity in the nth layer.

This equation relates the motions at the top and bottom of a layered system, incorporating the conditions of continuity in stress and velocity at each interface. In the particular case being considered, there is no up-coming shear wave in the nth layer, and the top of the layered system is bounded by a free surface. Following Haskell (1962), we then set

$$\omega_n'' = 0$$
$$\sigma_o = 0$$
$$\widehat{T}_o = 0$$

corresponding to the above conditions.

Under these conditions, we can solve the four algebraic equations which are implied in the single matrix equation for Δ_n', ω_n', $\overset{\bullet}{u}_o/c$, $\overset{\bullet}{w}_o/c$ in terms of Δ_n''. In particular, we can obtain

$$\frac{\overset{\bullet}{u}_o}{c\,\Delta_n^{''}} = 2(J_{32} - J_{42})\,/\text{Den}$$

and   (2)

$$\frac{\overset{\bullet}{w}_o}{c\,\Delta_n^{''}} = 2\,(J_{41} - J_{31})\,/\text{Den}$$

where the J's are the elements of the 4×4 matrix defined by $E_n^{-1}\,A_{n-1}\,\cdots\,A_1$ and $\text{Den} = (J_{11}-J_{21})(J_{32}-J_{42}) - (J_{12}-J_{22})(J_{31}-J_{41})$. Rather than use the ratios of $\overset{\bullet}{u}_o$ and $\overset{\bullet}{w}_o$ to $c\,\Delta_n^{''}$ as developed in equations (2), we shall find it more convenient to obtain the ratio of the components of the surface particle velocity to the total particle velocity in the nth layer due to the incident dilatational wave.

$$U_T = \sqrt{\overset{\bullet}{u}_n^{\,2} + \overset{\bullet}{w}_n^{\,2}} = \alpha_n \Delta^{''}$$ (3)

Therefore from equations (2) and (3) we have

$$\frac{\overset{\bullet}{u}_o}{V_T} = 2\,\frac{c}{\alpha_n}\;(J_{42}-J_{32})\,/\text{Den}$$

$$\frac{\overset{\bullet}{w}_o}{V_T} = 2\,\frac{c}{\alpha_n}\;(J_{41}-J_{31})\,/\text{Den}$$ (4)

These same equations represent the ratios of the surface displacements to the displacements at depth since we are considering steady-state simple harmonic motion. Thus we have

$$u_o/AT_n = \frac{2c}{\alpha_n} (J_{42} - J_{32})/\text{Den}$$

$$w_o/AT_n = \frac{2c}{\alpha_n} (J_{41} - J_{31})/\text{Den}$$

(5)

where AT_n is the amplitude of the total particle displacement of the incident dilatational wave in the nth layer. Equations (5) are, in effect, the equations for the transmission coefficients which relate the motion at the free surface of the layered system to the motion at the base of the system directly beneath the point on the surface. These coefficients are complex functions involving the frequency, the horizontal phase velocity (which is related to the angle of incidence), and the layer parameters. We shall use the symbols TU and TW as defined below for these coefficients.

$$TU = \frac{u_o}{AT_n}$$

(6)

$$TW = \frac{w_o}{AT_n}$$

In a later section we shall examine the values of TU and TW which were obtained by evaluating equations (5) for a number of crustal models. These models have such features of seismological interest as relatively thin low velocity layers both at the surface and at some depth, and relatively thick and relatively thin total crustal thicknesses. Before doing so, however, we will present some applications of TU and TW which are of interest.

Since TU and TW are dependent on crustal structure, it may be possible to use measurements of those quantities to determine crustal structure. If the input wave is of the form $(AT_n) e^{i(pt-kx)}$, then the surface motion can be represented as

$$u_o = (AT_n) (TU) e^{i(pt-kx)}$$

$$w_o = (AT_n) (TW) e^{i(pt-kx)}$$

(7)

Since every point on the free surface is executing the same type of motion, although with different phases, we can consider the point $x = o$ without any loss of generality. With this understanding, and since the system is linear we can express the surface motion due to an incident wave of the form

$$INCIDENT\ WAVE = \frac{1}{2\pi} \int_{-\infty}^{\infty} (AT_n) e^{ipt} dp$$

as

$$u_o(t)\Big|_{x=o} = \frac{1}{2\pi} \int_{-\infty}^{\infty} (AT_n)(TU) e^{ipt} dp$$

(8)

and

$$w_o(t)\Big|_{x=o} = \frac{1}{2\pi} \int_{-\infty}^{\infty} (AT_n)(TW) e^{ipt} dp$$

where AT_n is the frequency spectrum of the input wave, and may be a complex function of frequency.

In actual practice $u_o(t)$ and $w_o(t)$ are recorded by seismographs and from this motion we would like to obtain information about the crust, or possibly about the source. Upon examining equations (8) we see that

the effects of the crust and source are interconnected in the quantities we usually observe. In order to separate these two effects, let us consider the Fourier transforms of the ground motion. If we define the transform of ζ as $F(\zeta)$, we have

$$F(u_0(t)) = (AT_n) (TU) \tag{9}$$

$$F(w_0(t)) = (AT_n) (TW)$$

The left-hand side of equations (9) can be determined from the observations. Thus we can determine the product of the source spectrum and the transmission effect of the layers. However, if we take the ratio $F(u_0(t))/F(w_0(t))$ we obtain a quantity which is independent of the frequency content of the source.

$$\frac{F(u_0(t))}{F(w_0(t))} = \frac{TU}{TW} \tag{10}$$

This ratio depends on the frequency, the horizontal phase velocity (angle of incidence), and the layer parameters. We might look upon TU/TW as the tangent of an apparent angle of incidence which is a function of frequency.

Since this ratio depends on the structure, it should be possible to determine the structure, or the class of structures, which give theoretical ratios to match the ratios obtained from the observations at a given site.

Phinney (1964) studied the crust near Albuquerque, N. M., by matching curves by trial and error, and found that structures which

were in agreement with the known geology gave good results, while those which were radically different gave poor agreement. Thus even at this time, this method of determining crustal structure appears to hold some promise. In the future it is quite possible that computer inversion programs similar to those used in surface wave work may be used.

This method has the advantage of allowing the determination of crustal structure beneath a station from the observations at that single station alone. Further, it does not depend on the picking of a single event, but uses the series of events which are recorded at the surface due to the direct arrival of the plane wave plus all of the arrivals coming from crustal reflections and conversions. This use of time sectionsrather than specific events may be of considerable help in those cases where it is difficult to identify specific events.

Once the crustal structure beneath a station is known, then it should be possible to determine the frequency content of the wave which struck the base of the crust. Referring to equations (9), we see that

$$(\Delta T_n) \; = \; \frac{F(u_0(t))}{TU} \; = \; \frac{F(w_0(t))}{TW} \tag{11}$$

This would allow us to obtain a better description of the source. A technique such as this may prove to be especially valuable in shear wave investigations.

It should be emphasized that in these considerations there are some important restrictions which may severely limit the application of these techniques. For example, the attenuation of high frequencies

which will provide the greatest resolution, the crust at the source, the curvature of the real earth, and the necessity of having perfectly matched instruments, all have effects which are not considered here. At the present time, we can only point to the work of Phinney (1964) which indicates that these methods are applicable at least to the study of the coarse features of crustal layering.

In the remainder of this report, we will examine the theoretical variation of the transmission coefficients TU and TW with frequency, angle of incidence, and crustal model in order to obtain some understanding of the effects of these parameters.

3. Steady-State Transmission Coefficients

In order to study the variation of TU and TW with frequency, angle of incidence, and crustal structure, we wrote programs for the IBM 1620 and 7072 computer systems. The basic program in 1620 form is shown in Appendix I (the 7072 form of this program differs only in the input-output statements and the addition of a Fourier integral routine). The input to this program specifies the ranges of angle of incidence and frequency, and the thicknesses, velocities and densities of the layers of the crustal model to be considered. In this study, the angle of incidence was varied from 21 degrees to 53 degrees in steps of 4 degrees, and the frequency was varied from .02 cps to 10.0 cps in steps of .02 cps. These ranges were chosen because of their applicability in various seismological studies.

The models which were chosen are listed in Table I together with a brief description of their predominant features. We determined the values of the amplitude and phase of TU and TW for each of these models at the values of the angle of incidence and frequency given above. Due to the extent of the data (greater than 27,000 individual points), only a representative number of curves will be shown to illustrate the conclusions. The complete set of data is available in the form of punched cards at the St. Louis University Institute of Technology, Department of Geophysics. It should be noted, however, that unless the entire range of data is desired, it is often more convenient to compute the data in the desired range by using the program in

TABLE I

Model No.	Description of Main Feature	Thickness (Km.)	P- Velocity (Km/Sec)	S- Velocity (Km/Sec)	Density (Gm/Cm^3)
1.[*]	A crustal model similar to that of the central U.S. with a thin, low-velocity surface layer	1.0 19.0 18.0	4.40 6.20 6.40 8.20	2.50 3.50 3.70 4.60	2.70 2.80 2.90 3.30
2.[*]	The same model as 1, but the surface layer is removed and the second layer is extended to keep the total crustal thickness constant	20.0 18.0	6.20 6.40 8.20	3.50 3.70 4.60	2.80 2.90 3.30
3.	A crustal model with a relatively thin low-velocity layer at depth	15.0 5.0 18.0	6.20 6.00 6.40 8.20	3.50 3.30 3.70 4.60	2.80 2.80 2.90 3.30
4.	The model of Haskell (1962) included as a check on calculations	37.0	6.285 7.960	3.635 4.600	2.869 3.370
5.	A crustal model representing a relatively thick crust, having the same velocities as 2 and 6	30.0 20.0	6.20 6.40 8.20	3.50 3.70 4.60	2.80 2.90 3.30
6.	A crustal model representing a relatively thin crust, having the same velocities as 2 and 5	15.0 5.0	6.20 6.40 8.20	3.50 3.70 4.60	2.80 2.90 3.30

[*]Models 1 and 2 were suggested by Mr. T.V. McEvilly as corresponding to crustal structure in Missouri and/or the central United States.

Appendix I.

In discussing the figures on the following pages, we shall make use of the terms "character" and "shift." "Character" is intended to refer to the relations between the amplitudes for various frequencies for a given angle of incidence and a given model. That is, "character" refers to the general shape of adjacent peaks and troughs. On the other hand, "shift" is intended to refer to the fact that the frequency variation often appears to be compressed or extended and thus segments of the curves having similar character are displaced in the frequency domain.

In Figure 2, we see the variation of |TU| for model 6 plotted for six angles of incidence in the frequency range from 0.02 to 2.0 cps. This particular model was chosen because it illustrates features of the curves which are found in all of the models, and in addition, the frequency variation is slow enough that the features may be readily observed.

In all of the figures of this type, the variation with frequency is obvious. With few exceptions, it has not been possible to interpret the frequency variations of these curves as being due to any particular physical phenomenon (for example, multiply reflected waves of a specific type within the whole crustal model or any section). In this figure, we can observe the effect of the angle of incidence on the frequency variation. We can make the following observations:

1) As the angle of incidence increases, the average amplitude increases. This might be expected in view of the fact that the

FIGURE 2. THE EFFECT OF THE ANGLE OF INCIDENCE ON THE FREQUENCY VARIATION OF |TU| FOR MODEL 6.
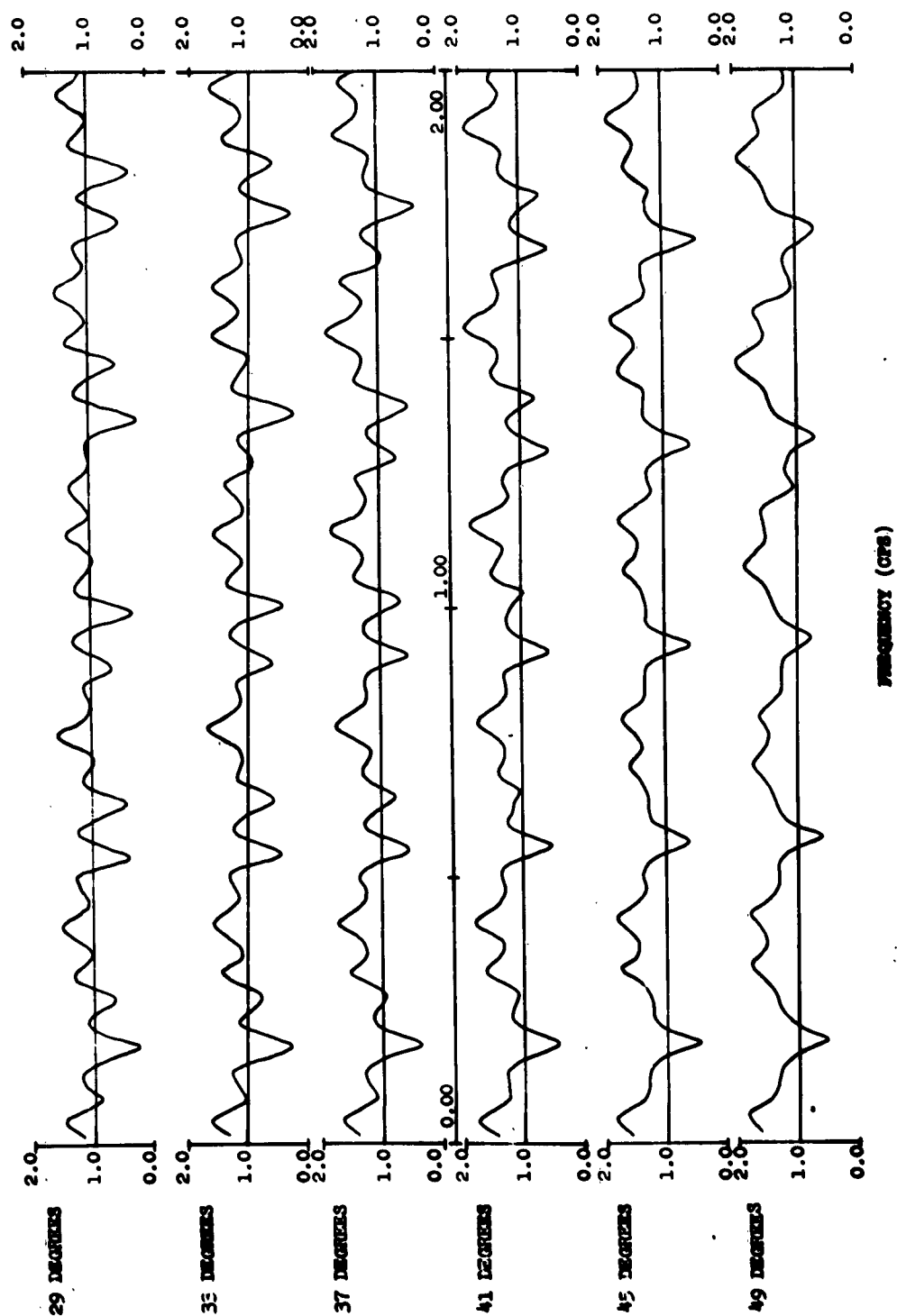
dilatational input will have a greater horisontal component at the larger angles of incidence.

2) The frequency variation is more rapid at small angles of incidence than at larger angles. This is due to the fact that for the small angles of incidence in this range, the P to S conversion has a large horisontal component which effectively doubles the number of wave types causing interference in the physical realm.

3) As the angle of incidence increases, the character of the variation changes considerably. For example, the peak which occurs at about 0.14 cps in the graph for the 29 degree angle of incidence has almost disappeared at 49 degrees. On the other hand, some relatively large amplitudes at large angles were relatively minor at smaller angles. However, the extremes, that is the largest maxima or smallest minima, tend to persist and this suggests their dependence on the dilatational energy.

4) As the angle of incidence increases, we can observe a systematic shift. At low frequencies, this shift is not so apparent, but at the higher frequencies, it becomes progressively more evident. The rate of shift appears to increase continuously with the increase in angle of incidence, if allowances for the change in character are made. It will be seen in later figures that a decrease in the velocity also can cause this shift to take place.

In Figure 3, we see the variation of $|TW|$ for model 6. In this figure, we can make the following observations:

FIGURE 3. THE EFFECT OF THE ANGLE OF INCIDENCE ON THE FREQUENCY
VARIATION OF |TM| FOR MODEL 6
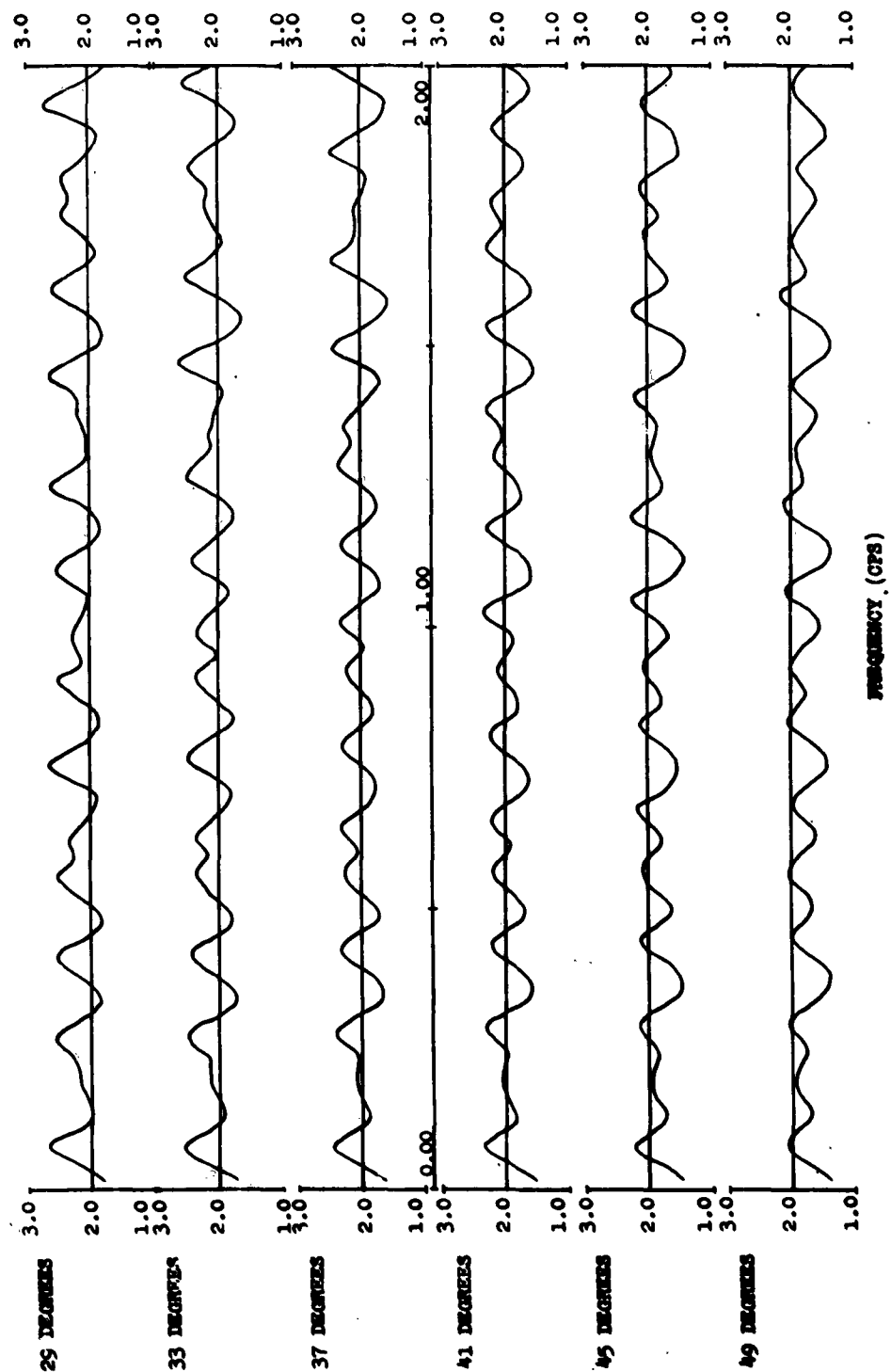
FREQUENCY (CPS)

1) As the angle of incidence increases, the average amplitude decreases. This again would be expected on the basis of the vertical component of the dilatation being smaller at large angles of incidence.

2) The variation with frequency is smaller at large angles of incidence than it is at small angles. This corresponds to the fact that for large angles of incidence the vertical component of P is small, and further, the angle of incidence of the converted S motion is such that the S motion has only a small vertical component even when the angle of incidence of the dilatational energy at the base of the crust is 53 degrees.

3) As before, the character of the variation changes considerably when the angle of incidence increases. For example, the trough which appears in the range 0.14 to 0.16 cps at an angle of 29 degrees has become a peak at 49 degrees.

4) Again as the angle of incidence increases we can observe a shifting of the curves. This shift increases with both angle and frequency.

In comparing Figures 2 and 3, we note that for small angles the average amplitudes in Figure 2 are smaller than those in Figure 3. At larger angles, the two are nearly equal. The frequency variation of the $|TU|$ is more irregular than that of $|TW|$. The shifting effect is of the same magnitude in both figures.
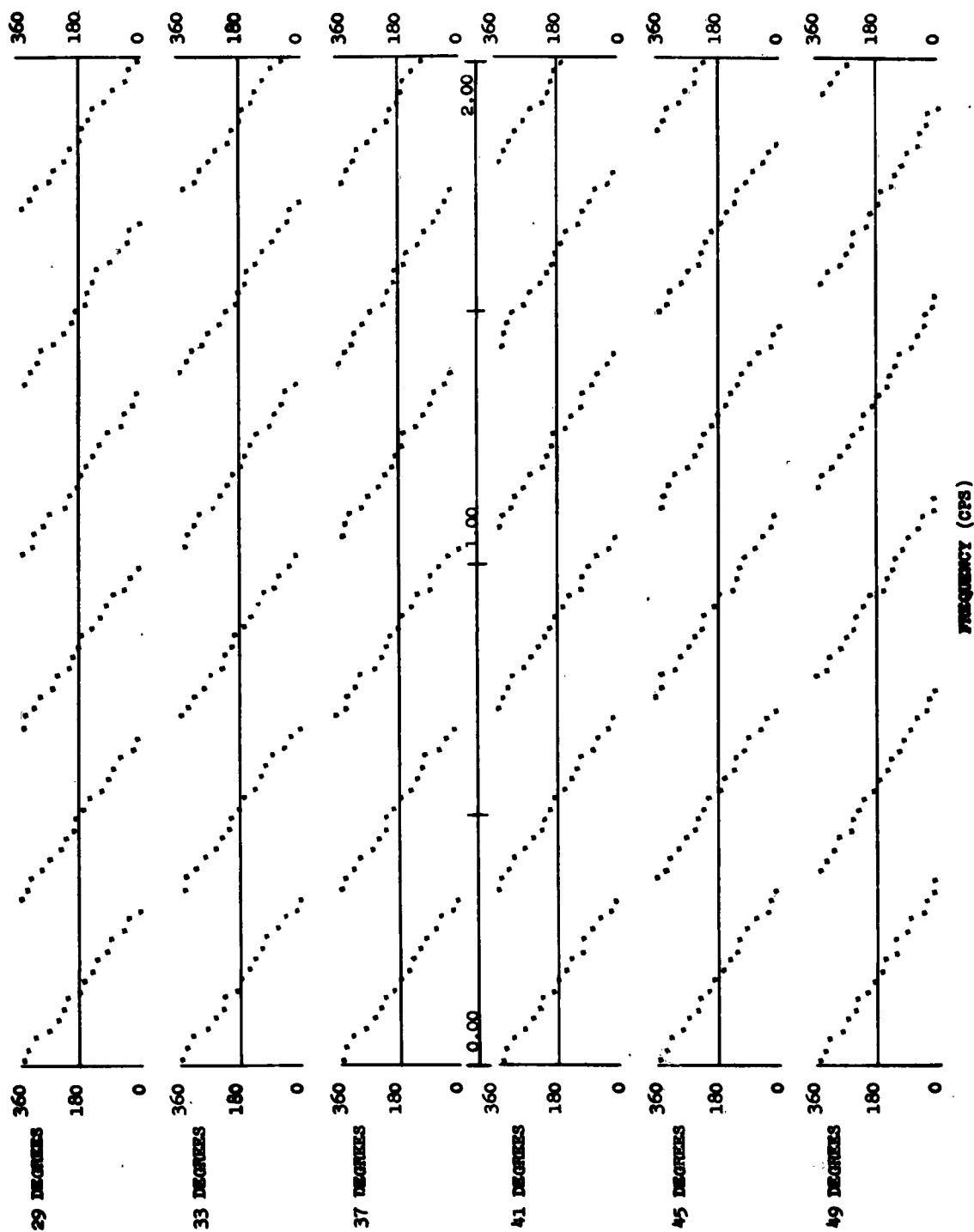
In Figure 4, we see the variation of the phase of TU for model 6, for a variety of angles in the same frequency range as above. For this figure, we can make the following observations:

FIGURE 4. THE EFFECT OF THE ANGLE OF INCIDENCE ON THE FREQUENCY
VARIATION OF THE PHASE OF TU FOR MODEL 6
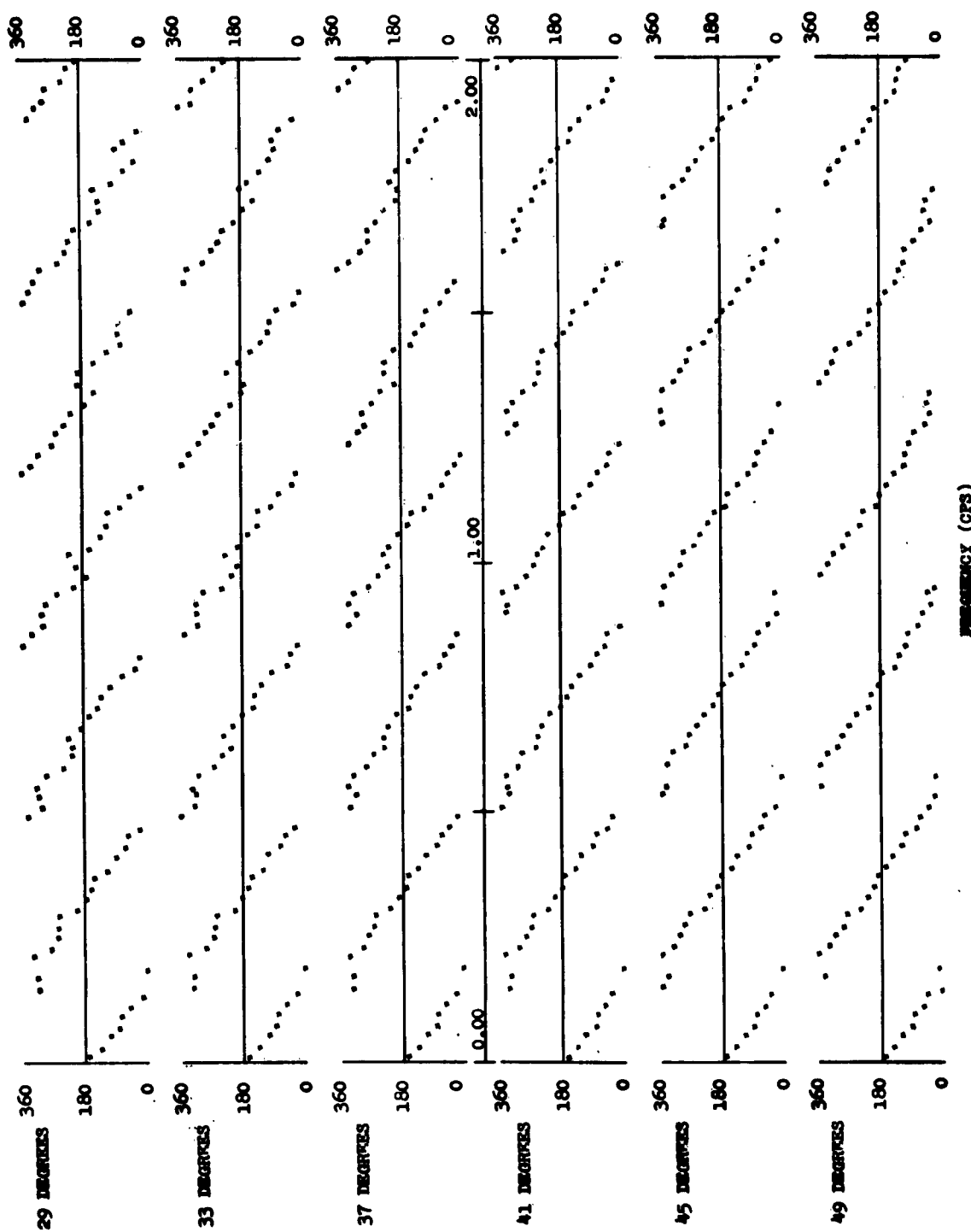
1) At very low frequencies, the phase shift approaches 180 degrees. This is the result of using the absolute value of the amplitude in the ratio rather than the horizontal component which would have a sign connected with it. If we allow for this change in phase caused by the mathematics, we can interpret this phase shift as saying that at low frequencies the incoming wave does not "see" the crust.

2) The phase shift is not linear, and the non-linearity is especially severe at small angles of incidence, where it often amounts to variations of up to 40 degrees. At larger angles, the phase shifts become very nearly linear in certain regions. As the frequency increases it appears that the phase shift becomes more irregular. However, as we will see in later figures, this is not progressive, but is a periodic phenomenon. The decrease of the non-linearity with increasing angle of incidence and the appearance of the vertical phase shifts, which will be considered next, both indicate that the non-linearity is due to the S conversions. However, we have been unable to interpret this on the basis of a particular ray arrival or the interference of particular ray events.

3) As the angle of incidence increases, the absolute value of the slope of the curves becomes smaller. This is related to the shift which appeared in the amplitudes of TU and TW . We can interpret this decrease in the absolute value of the slope as the angle of incidence increases in terms of the travel time of the wave front. TU and TW represent the relations between two points, one at the base of the crust, and the other at the free surface directly above the first

point. As the angle of incidence increases, the distance between the two points along the normal to the wave front decreases. If we decrease the distance, then the time required for the wave front to move through these two points decreases, and the phase difference between two frequencies differing by an amount Δ p decreases.

Phase difference = $(\Delta p)t$

where t = travel time.

This explanation was suggested from a consideration of plane waves; and in the steady-state case, the surface of constant phase may be a curve due to the superposition of the many reflections and conversions possible. However, since this same argument (i.e., based on travel times) explains the shifts in the amplitude variation and the changes in the slope of the phase variation observed in the various models, we believe that this explanation is essentially correct.

In Figure 5, we have the variation of the phase of TW for model 6 for a series of angles in the same frequency range as above.

1) At low frequencies, the phase shift approaches 360 degrees. Again allowing for the sign convention and the sign of the total amplitude at depth, this 360 degree phase shift at low frequencies indicates that the low frequencies are not affected by the crust.

2) The phase shift for TW is very nearly linear, and the extent of the non-linearity does not increase appreciably as the angle of incidence increases in this range.

3) As the angle of incidence increases, the absolute value of the slope becomes smaller. This is due to the same causes as were

FIGURE 5. THE EFFECT OF THE ANGLE OF INCIDENCE ON THE FREQUENCY
VARIATION OF THE PHASE OF TV FOR MODEL 6

previously mentioned for the phase of TU.

In Figures 6, 7, and 8, we have the variation in the frequency dependence of $|TU|$ as a function of the model. In Figure 6, the variation is plotted for an angle of incidence of 21 degrees, Figure 7 for 33 degrees, and Figure 8 for 45 degrees. The observations which will be made apply to all three figures.

1) The effect of the low-velocity surface layer in model 1 is to introduce a "long period," large amplitude oscillation in the frequency variation of $|TU|$ for this model.

Further, the low velocity in this surface layer causes a shift to the left in model 1 with respect to model 2 (which is identical except for this layer). This shift can be observed at frequencies greater than 1.5 cps.

2) Were it not for the long period variation and the shift caused by the difference in velocities, models 1 and 2 would have essentially the same frequency variation since the character of the two is quite similar.

3) Models 2 and 3 are similar in character at low frequencies, and although the character changes at higher frequencies, it might prove to be practically impossible to distinguish between these two models at frequencies less than 0.2 cps.

4) Models 3 and 4 are similar in amplitude, but the fine character is different. This is attributable to the differences in velocities. The shift between these two models is noticeable and is due to the one kilometer difference in the thickness of the total crust.
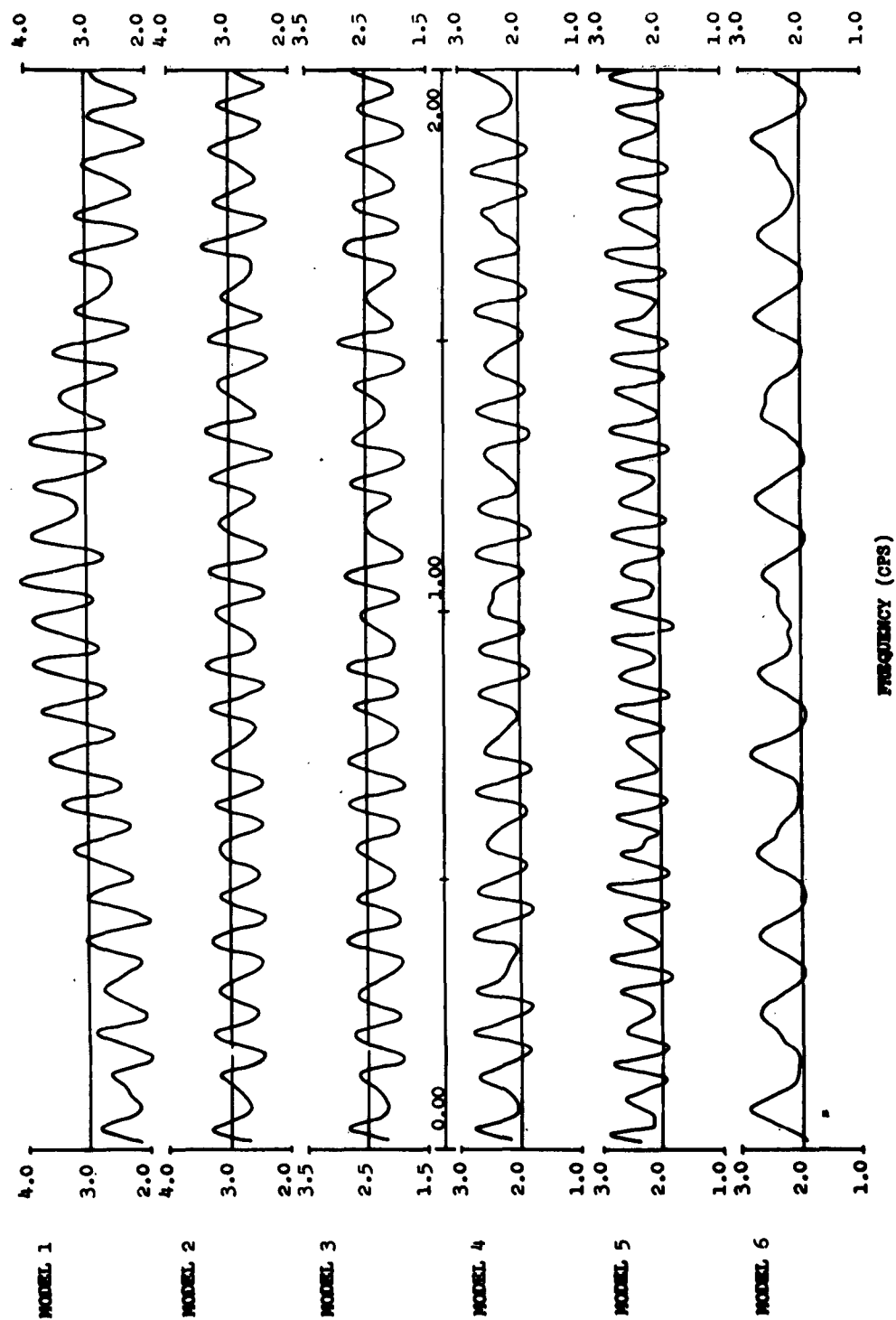
FIGURE 6. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF |TU| AT AN ANGLE OF INCIDENCE OF 21 DEGREES
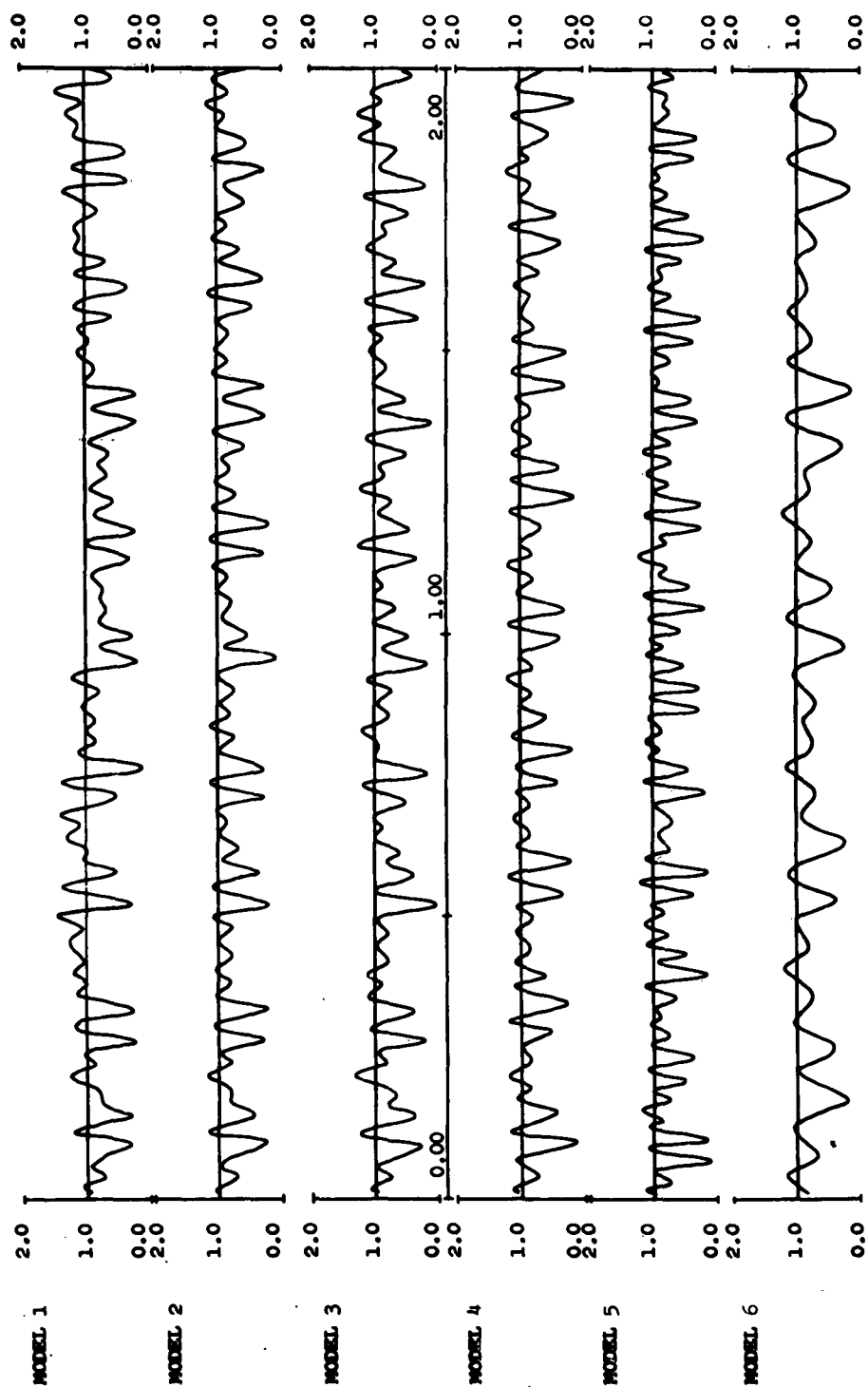
FIGURE 7. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION
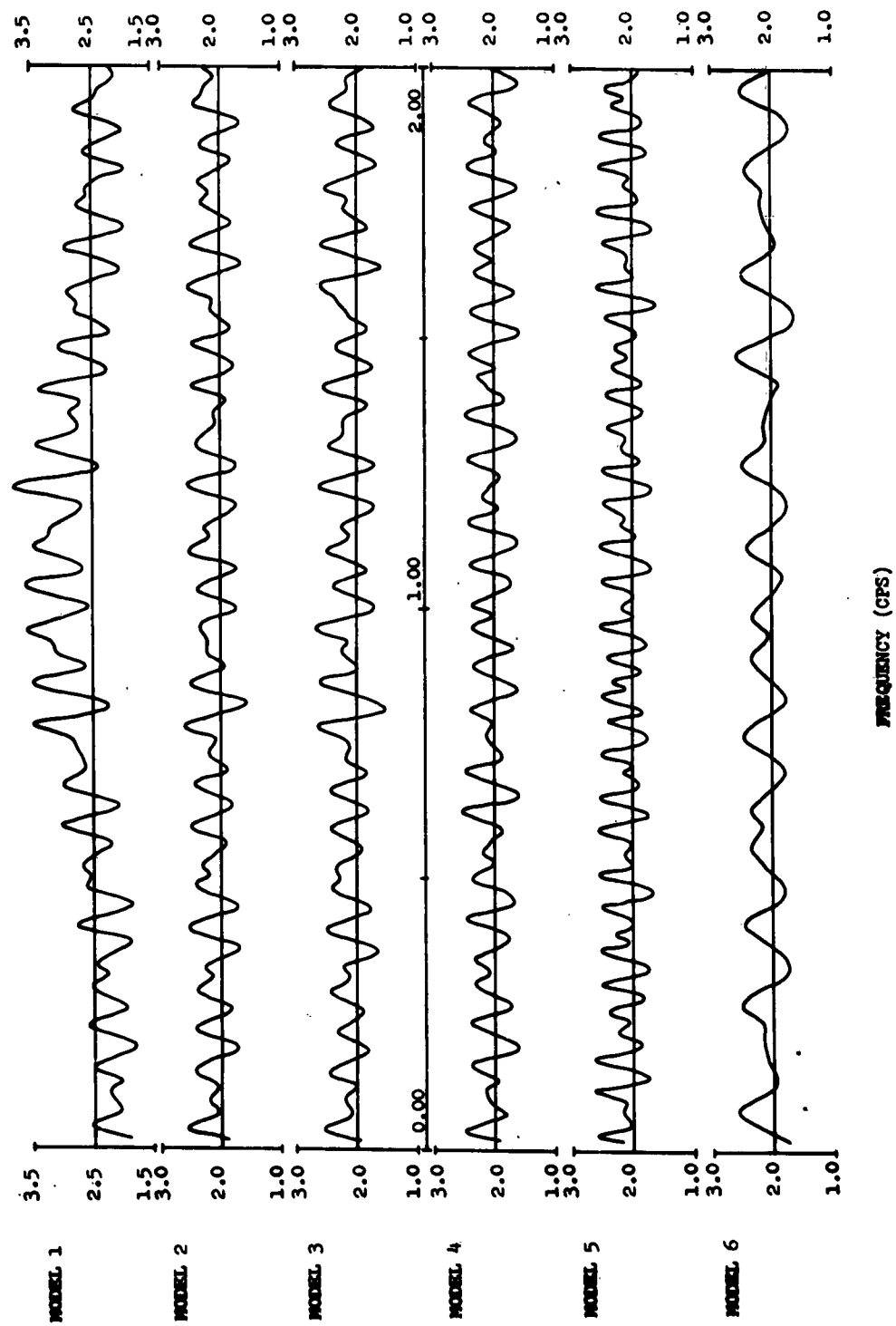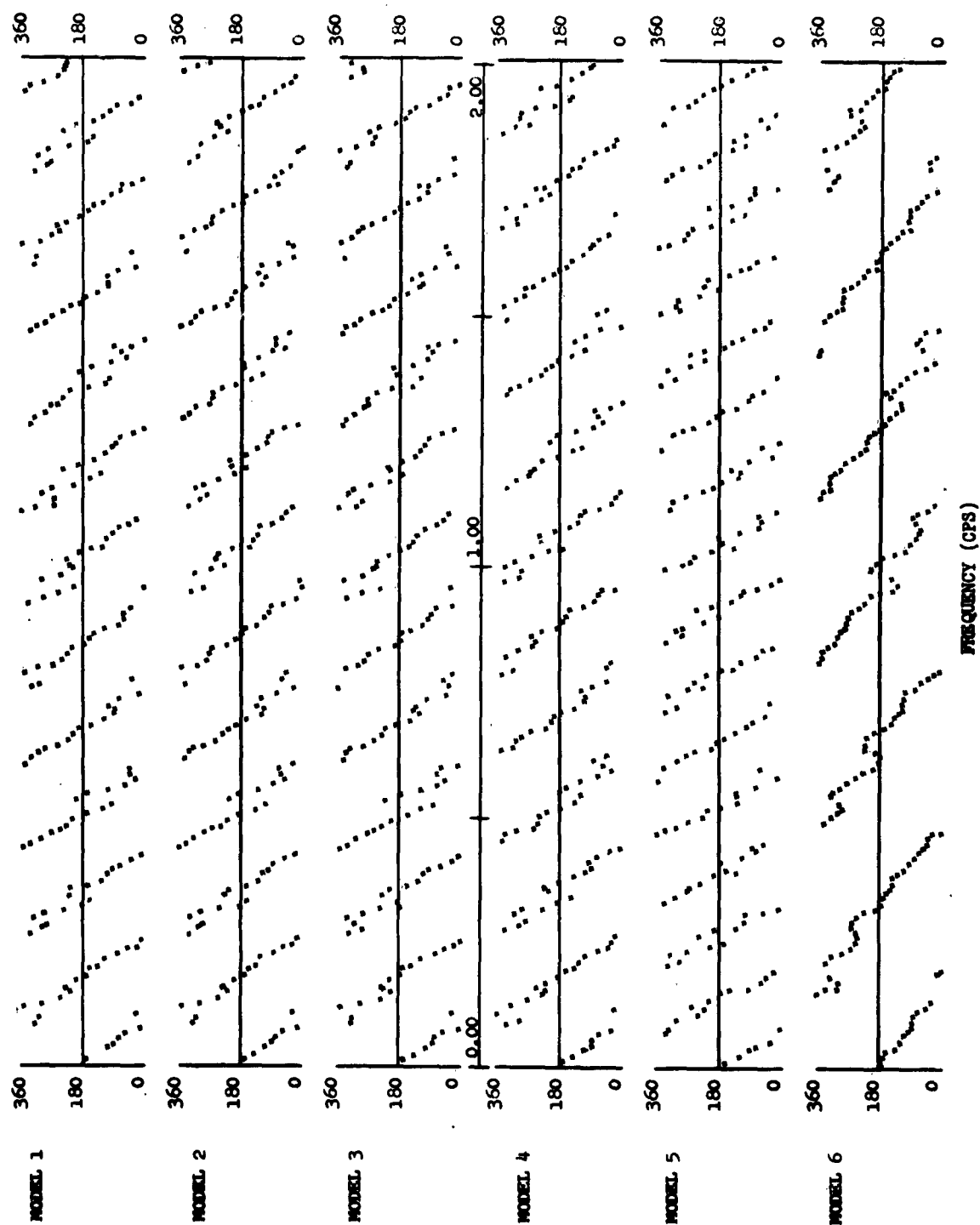OF |PU| AT AN ANGLE OF INCIDENCE OF 33 DEGREES
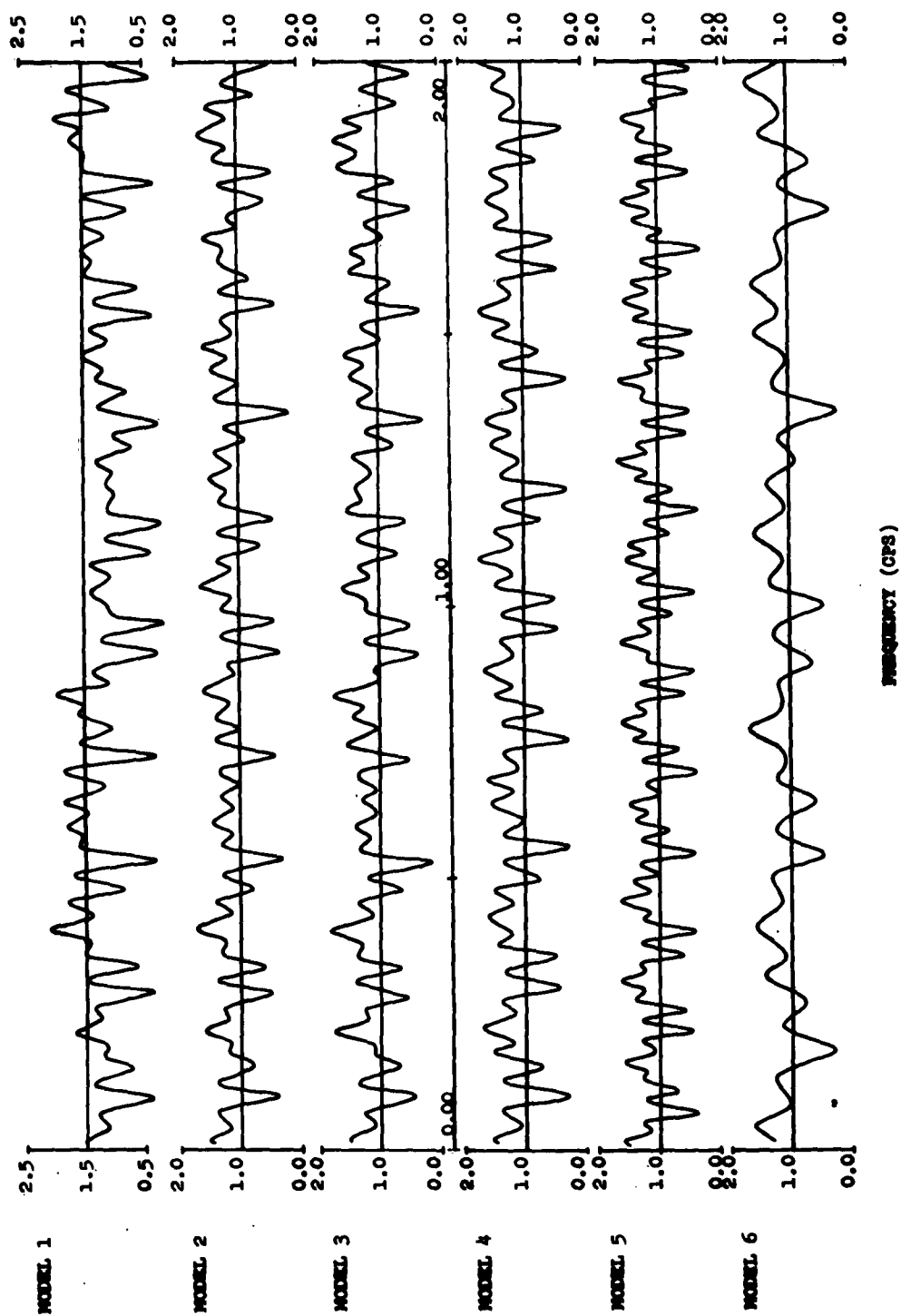
FREQUENCY (CPS)
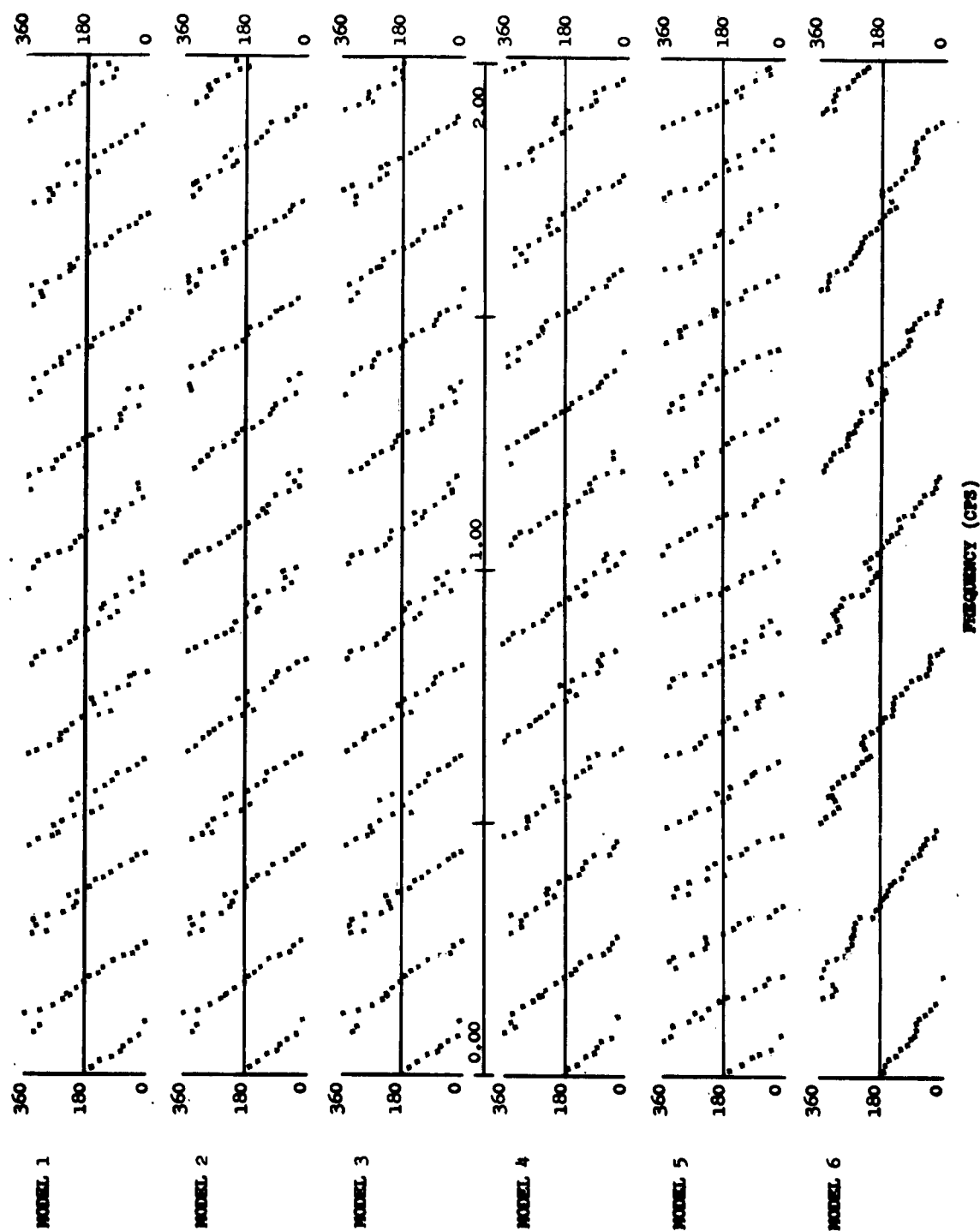
FIGURE 8. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION
OF |FU| AT AN ANGLE OF INCIDENCE OF 45 DEGREES
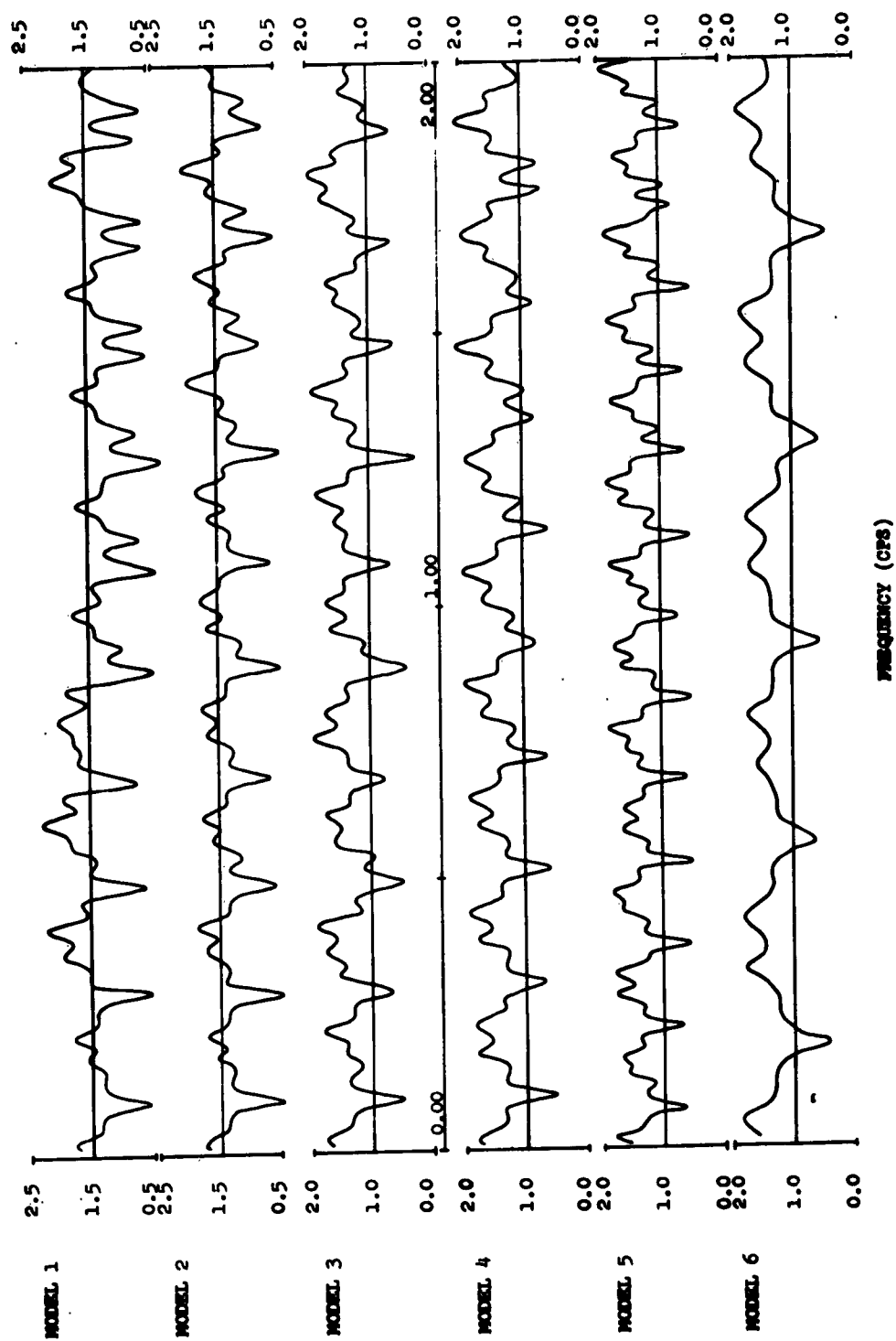
5) Model 5 has the same velocities as models 2 and 6 although the relative thicknesses of the layers are different. The character is actually quite similar in these three models. However, the shift to the left in model 5 due to the relatively large total crustal thickness is so great as to cause the amplitudes at a given frequency to be considerably different. At higher frequencies the character also changes with respect to that of model 2, but it still remains similar.

6) Model 6 is similar to model 5 in terms of the velocities,but is much thinner (20 kilometers in total thickness in model 6 versus 50 kilometers in model 5). This causes a considerable shift to the right in the values of $|TU|$ in model 6.

7) The difference in the models at a given low frequency is more apparent at large angles of incidence than at small angles. This is due to the change in character, the change in amplitude, and the shift. On the other hand, at the larger angles the character of the curves is relatively simple. The character alone does not give a good basis for distinguishing between the models, but the values of the co-efficients would have to be used.

In Figures 9, 10, and 11, we have relations similar to those shown in Figures 6, 7 and 8; however, in this case we are considering $|TW|$. The same comments as were made about $|TU|$ are applicable here. In comparing the two sets of curves, we see that the positions of the maxima and minima of the two sets of curves do not occur at the same place, and the characters of the two sets are entirely different.

FIGURE 9. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF |TM| AT AN ANGLE OF INCIDENCE OF 21 DEGREES

FIGURE 10. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF |TW| AT AN ANGLE OF INCIDENCE OF 33 DEGREES

FIGURE 11. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF RTM AT AN ANGLE OF INCIDENCE OF 45 DEGREES

34
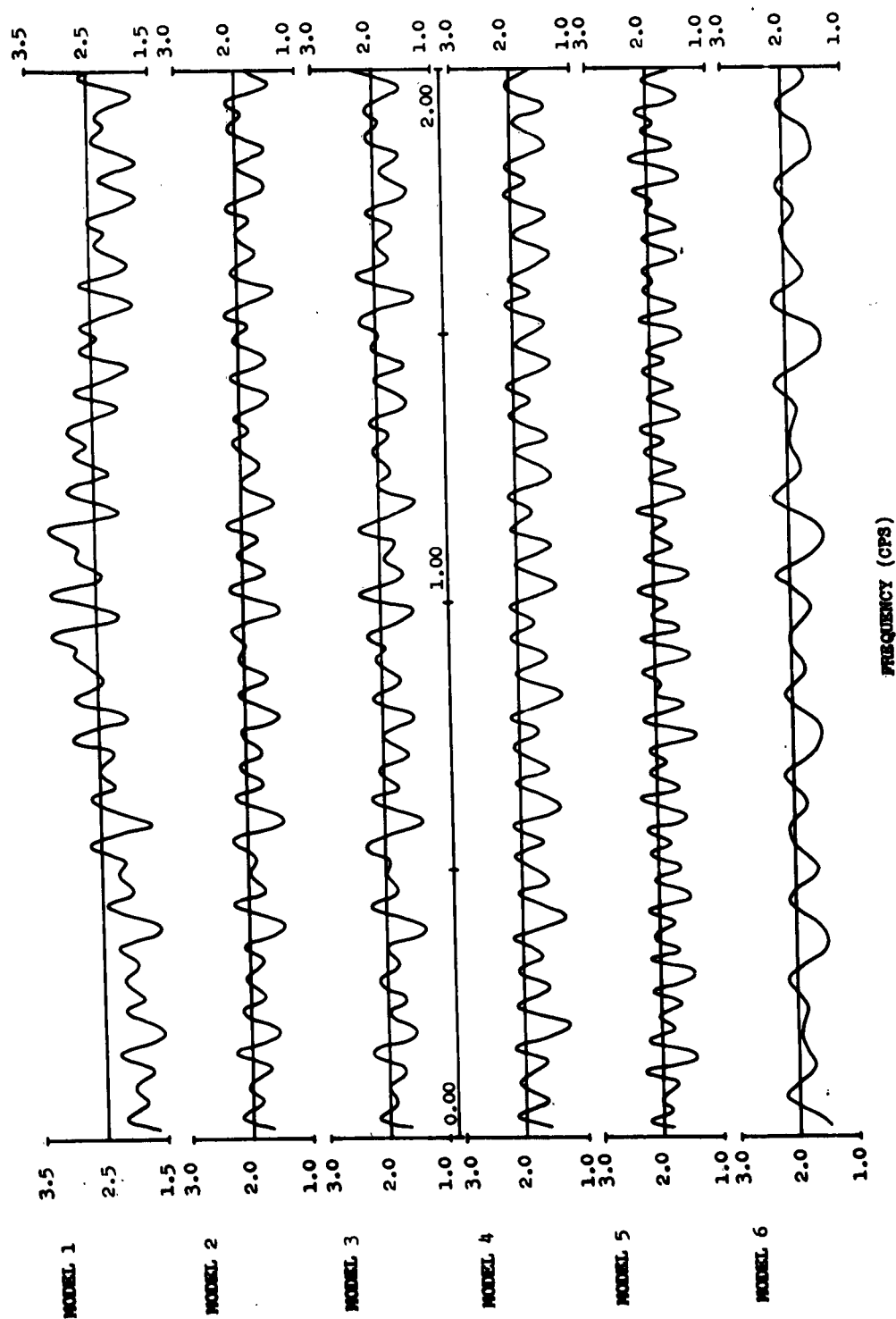
In Figures 12 through 14, we have the variation in the phase of TU for the various models at three angles of incidence. In these figures we can observe the following points:

1) The phases of the first three models at low frequencies are nearly the same.

2) The relations between the absolute values of the slopes of the models are model 6 < model 4 < model 3 < model 1 < model 2 < model 5.

3) The difference in the slopes becomes more apparent at the larger angles of incidence.

4) There are definite regions of non-linearity. The character of the non-linearity appears to be similar in all of the models. In these regions there appear to be discontinuities. However, on the basis of model 6 we would expect the variation to be continuous although very rapid.

In Figures 15-17, we have the variation in the phase of TW for the same models at the same angles of incidence. Here we can make the following observations:

1) There are slight variations in the linearity which become more apparent at the larger angles. However, for most practical purposes we could consider these phase shifts to be linear.

2) At low frequencies the phases of the first three models are nearly the same.

3) The values of the slopes are arranged in the same order as were the phases of TU. The slopes of the phase shifts of TW are very

FIGURE 12. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF THE PHASE OF TU AT AN ANGLE OF INCIDENCE OF 21 DEGREES

FREQUENCY (CPS)

MODEL 1 MODEL 2 MODEL 3 MODEL 4 MODEL 5 MODEL 6

FIGURE 13. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF THE PHASE OF TU AT AN ANGLE OF INCIDENCE OF 33DEGREES

FIGURE 14. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF THE PHASE OF TU AT AN ANGLE OF INCIDENCE OF 45 DEGREES
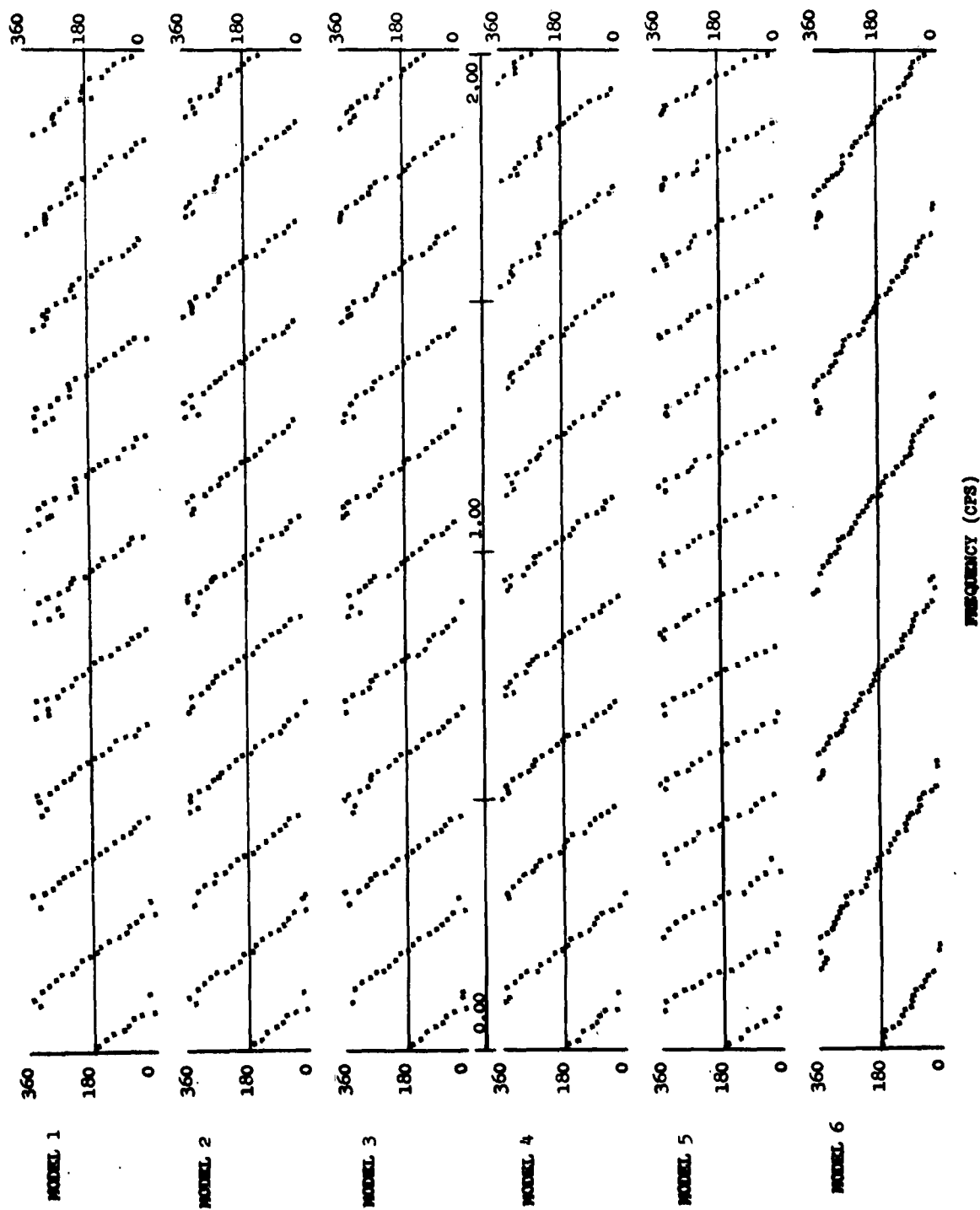
FIGURE 15. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION
OF THE PHASE OF TW AT AN ANGLE OF INCIDENCE OF 21 DEGREES
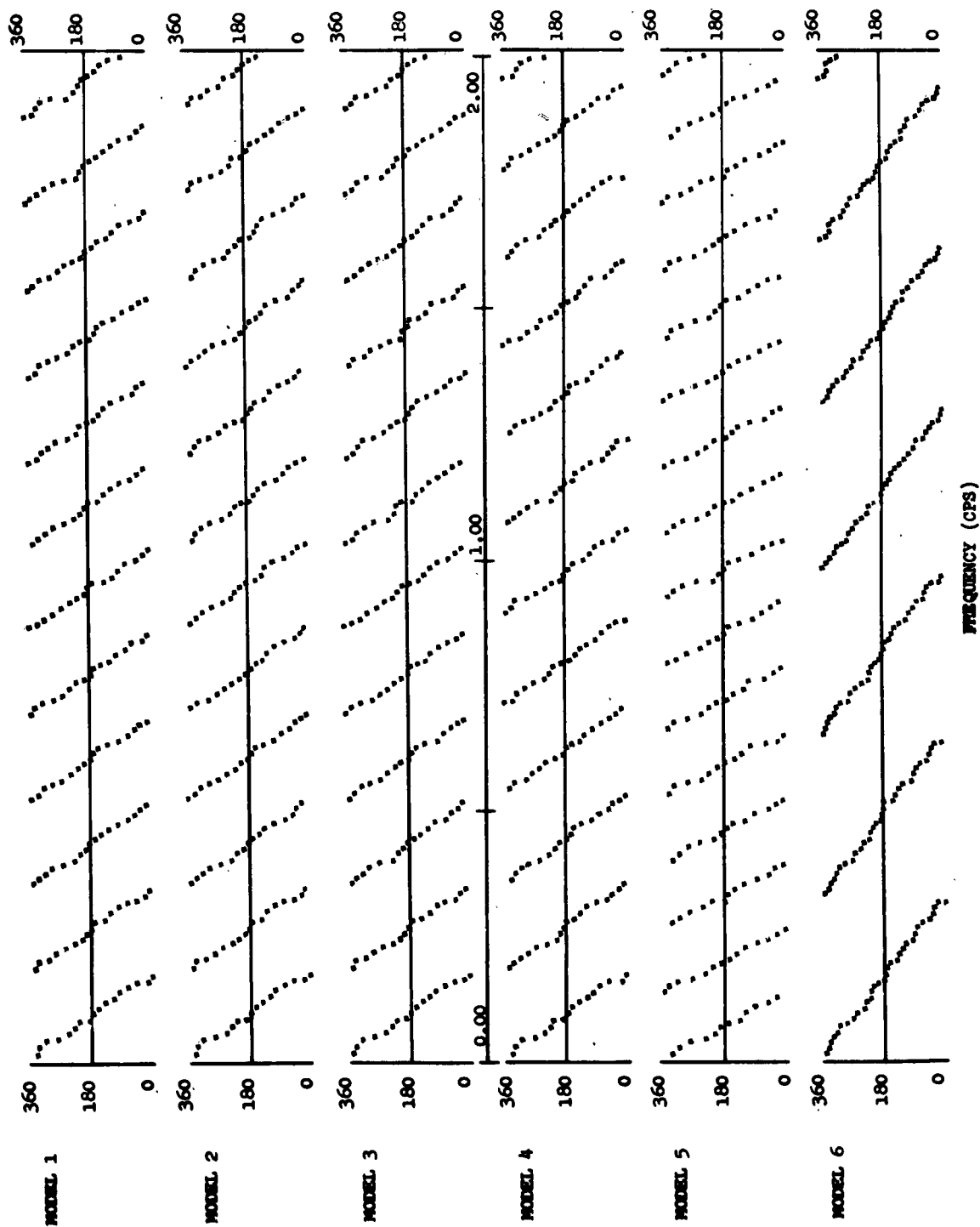
FIGURE 16. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF THE PHASE OF TW AT AN ANGLE OF INCIDENCE OF 33 DEGREES

MODEL 1

MODEL 2

MODEL 3

MODEL 4

MODEL 5

MODEL 6
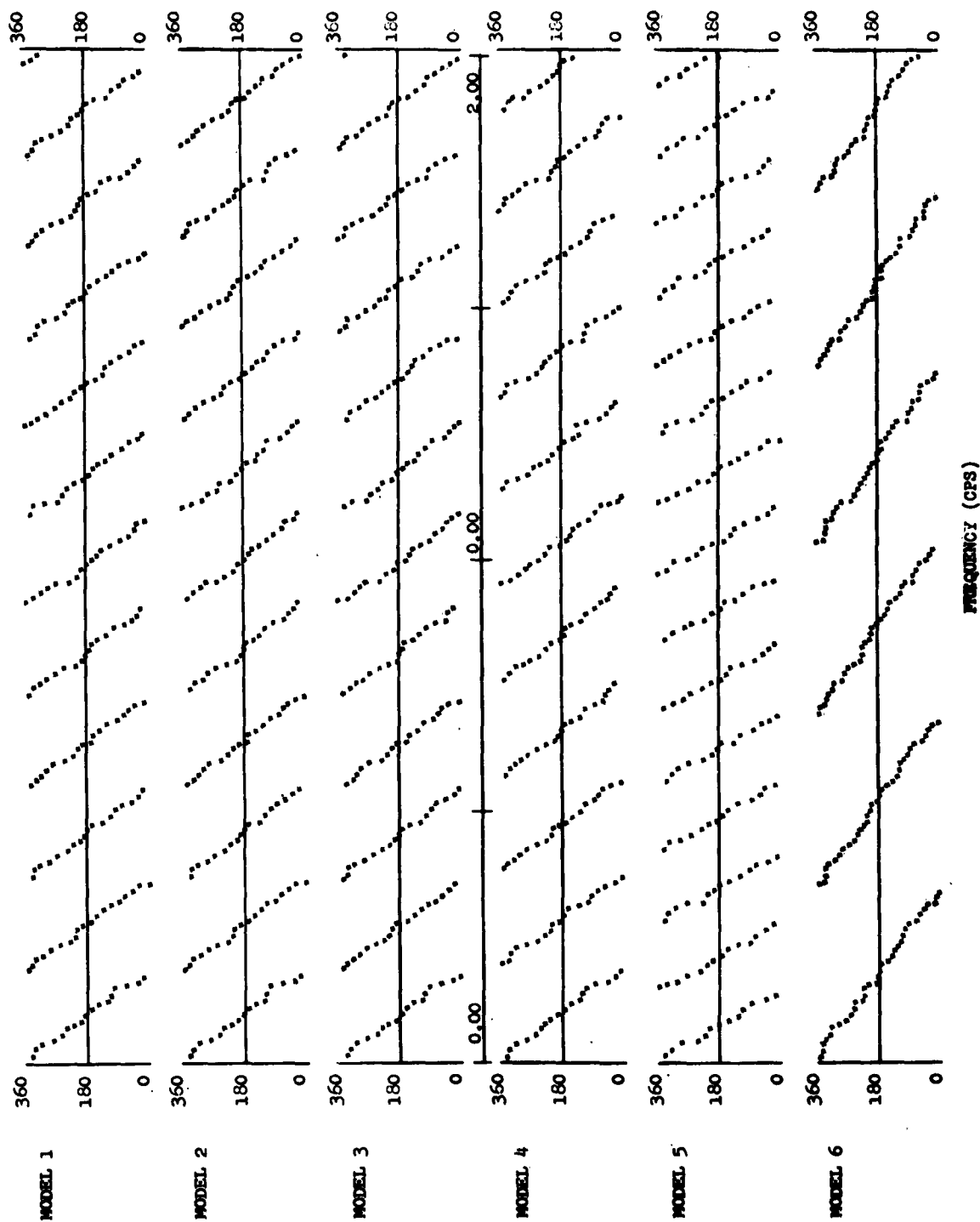
FREQUENCY (CPS)

FIGURE 17. THE EFFECT OF THE CRUSTAL MODEL ON THE FREQUENCY VARIATION OF THE PHASE OF TW AT AN ANGLE OF INCIDENCE OF 45 DEGREES
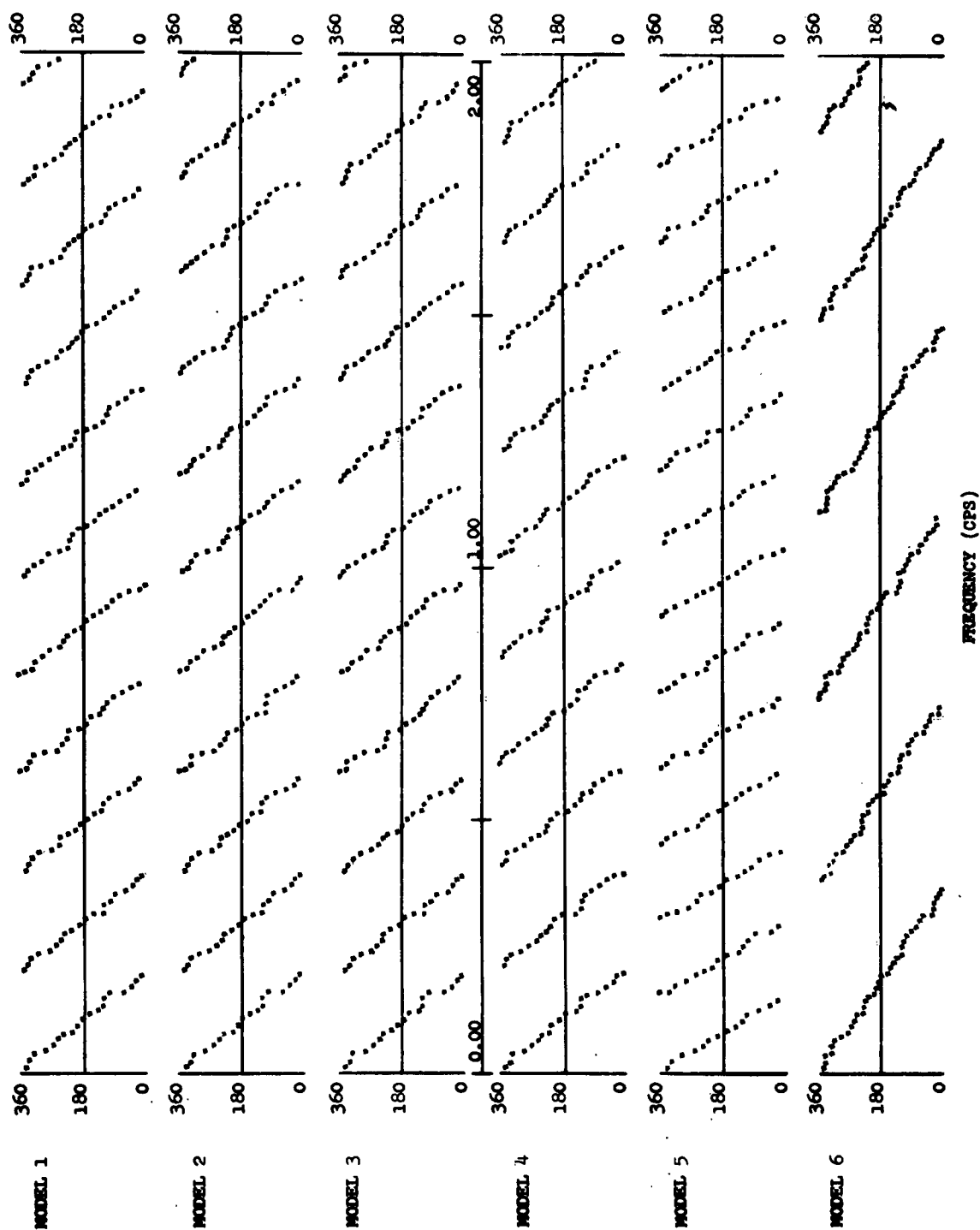
similar to the slopes of the corresponding phase shifts of TU.

In Figures 18-21 we see the variation of $|TU|$, $|TW|$, the phase of TU and the phase of TW, respectively, for all of the models for an angle of incidence of 33 degrees in the frequency range 2.02 to 4.00 cps. Upon comparing these figures with the previous figures, we see that the character is essentially the same as the corresponding figures in the lower frequency range.

One noticeable exception to this statement is especially prominent in Figure 20 in the frequency range from 2.02 to 2.5 cps for model 1. Here there is a region in which the phase changes very rapidly. This condition dies out but does repeat itself at higher frequencies. We see that the character of the amplitudes is also somewhat different in this range (Figures 7, 10); however, this is not so noticeable. This condition is apparently related to rotational interference since the unusual character appears mainly in the horizontal component.

Also we notice that the shift has increased to the point where it is no longer possible to correlate peaks and troughs. The values of TU and TW may be considerably different at a given frequency in this higher frequency range. From this we can conclude that our best opportunity for distinguishing between models would be to know TU/TW for the range of frequency from 0 to 3.0 cps at intervals of .015 cps. This would allow us to distinguish between models as similar as the first four in this paper.

In order to test the validity of the computations and to provide a comparison with conventional ray theory in the real time domain, the

FIGURE 21. THE FREQUENCY VARIATION OF THE PHASE OF TW IN THE FREQUENCY RANGE
FROM 2.02 TO 4.00 CPS AT AN ANGLE OF INCIDENCE OF 33 DEGREES
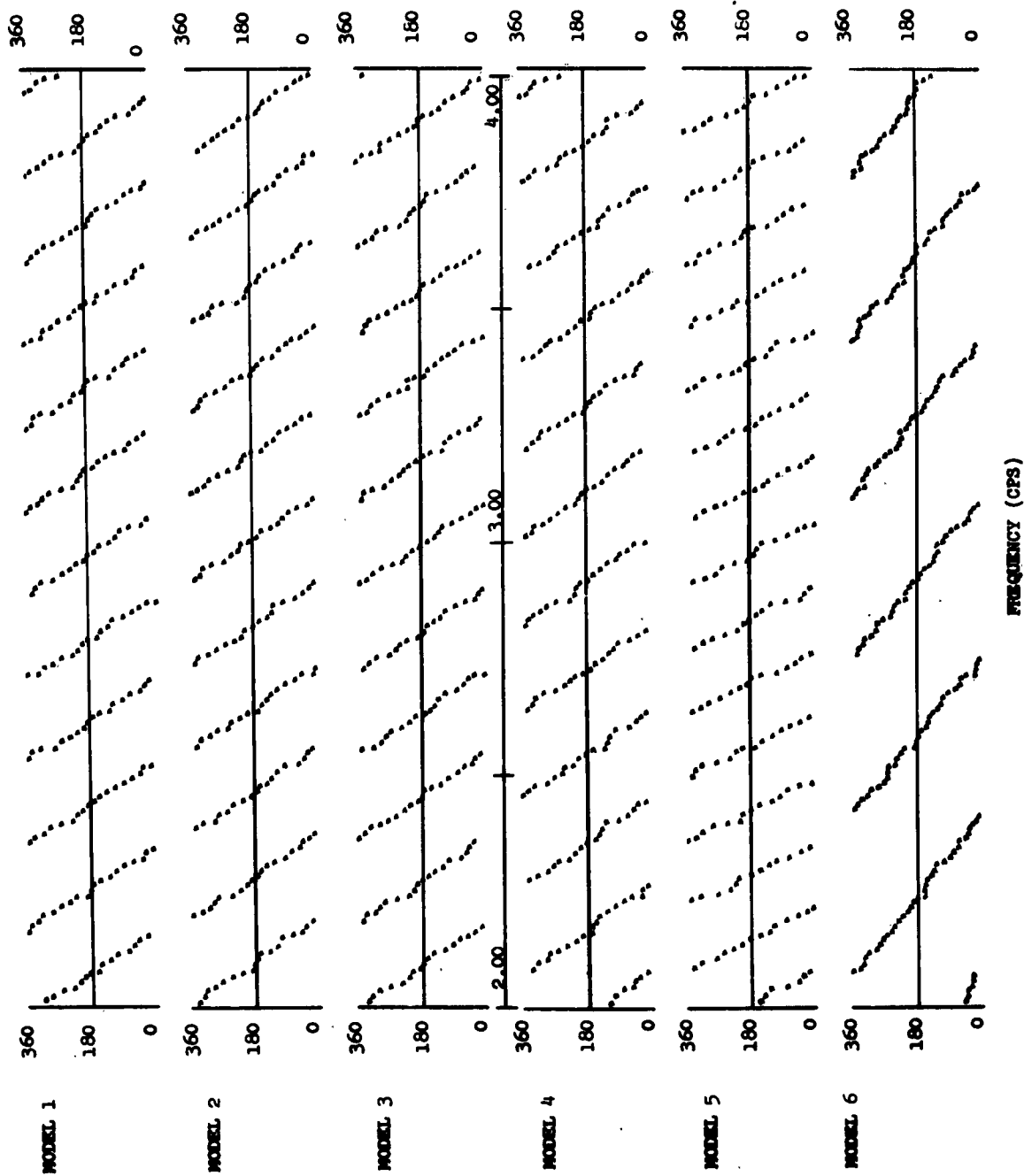
FREQUENCY (CPS)

FIGURE 20. THE FREQUENCY VARIATION OF THE PHASE OF TU IN THE FREQUENCY RANGE FROM 2.02 TO 4.00 CPS AT AN ANGLE OF INCIDENCE OF 33 DEGREES
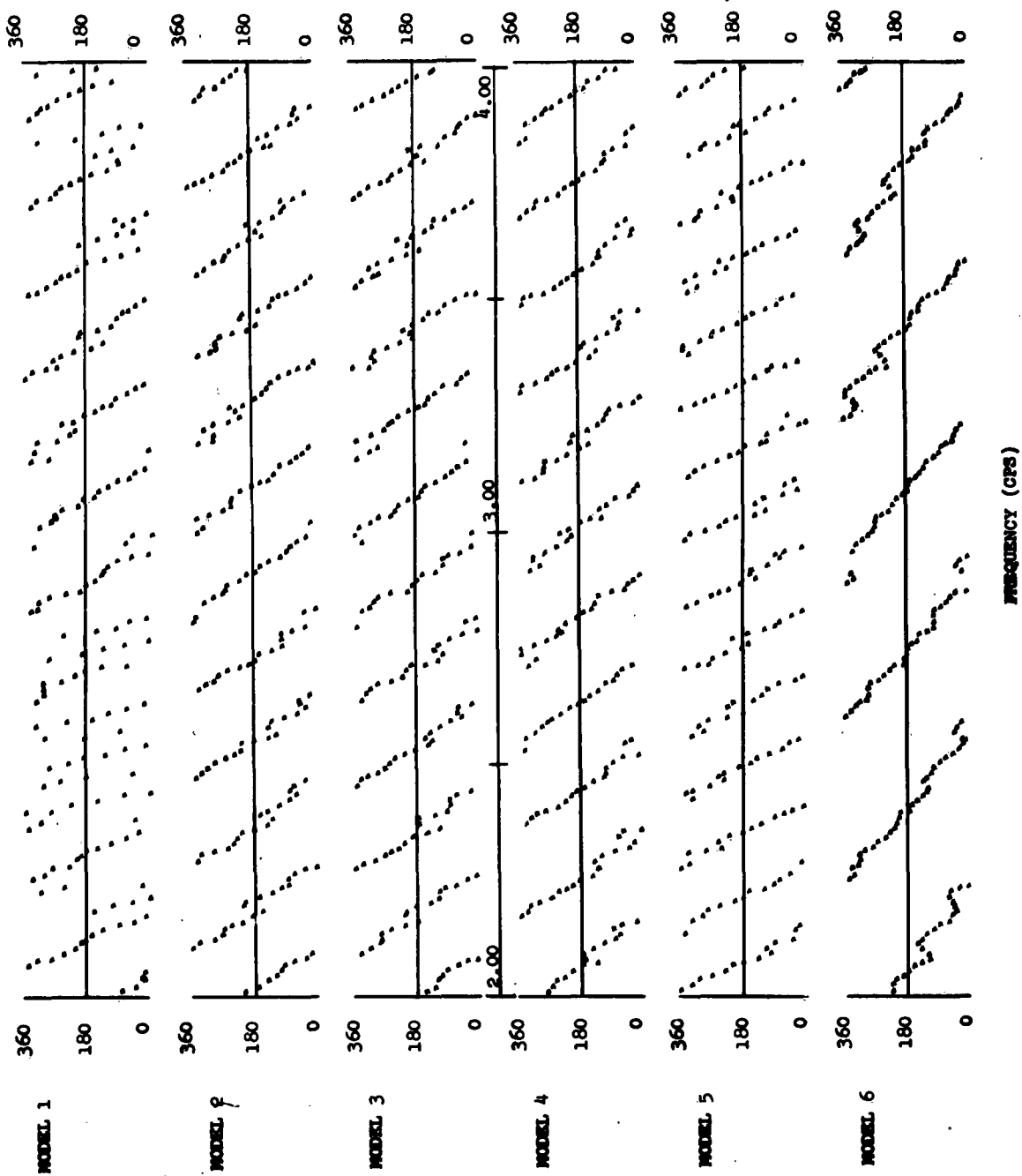
FREQUENCY (CPS)

FIGURE 19. THE FREQUENCY VARIATION OF RFM? IN THE FREQUENCY RANGE
FROM 2.02 TO 4.00 CPS AT AN ANGLE OF INCIDENCE OF 33 DEGREES
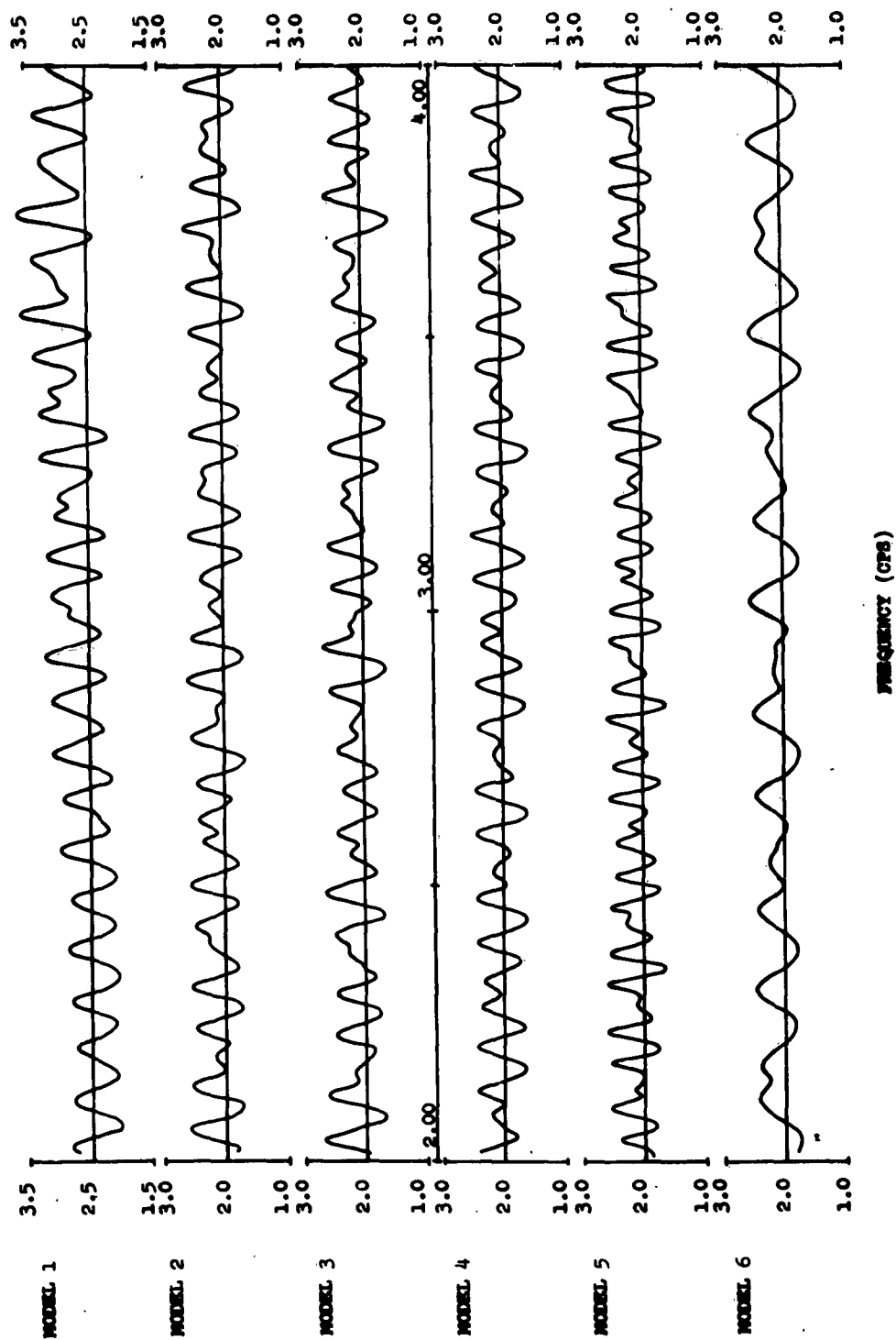
FREQUENCY (CPS)

FIGURE 18. THE FREQUENCY VARIATION OF |τu| IN THE FREQUENCY RANGE
FROM 2.02 TO 4.00 CPS AT AN ANGLE OF INCIDENCE OF 45 DEGREES

FREQUENCY (CPS)
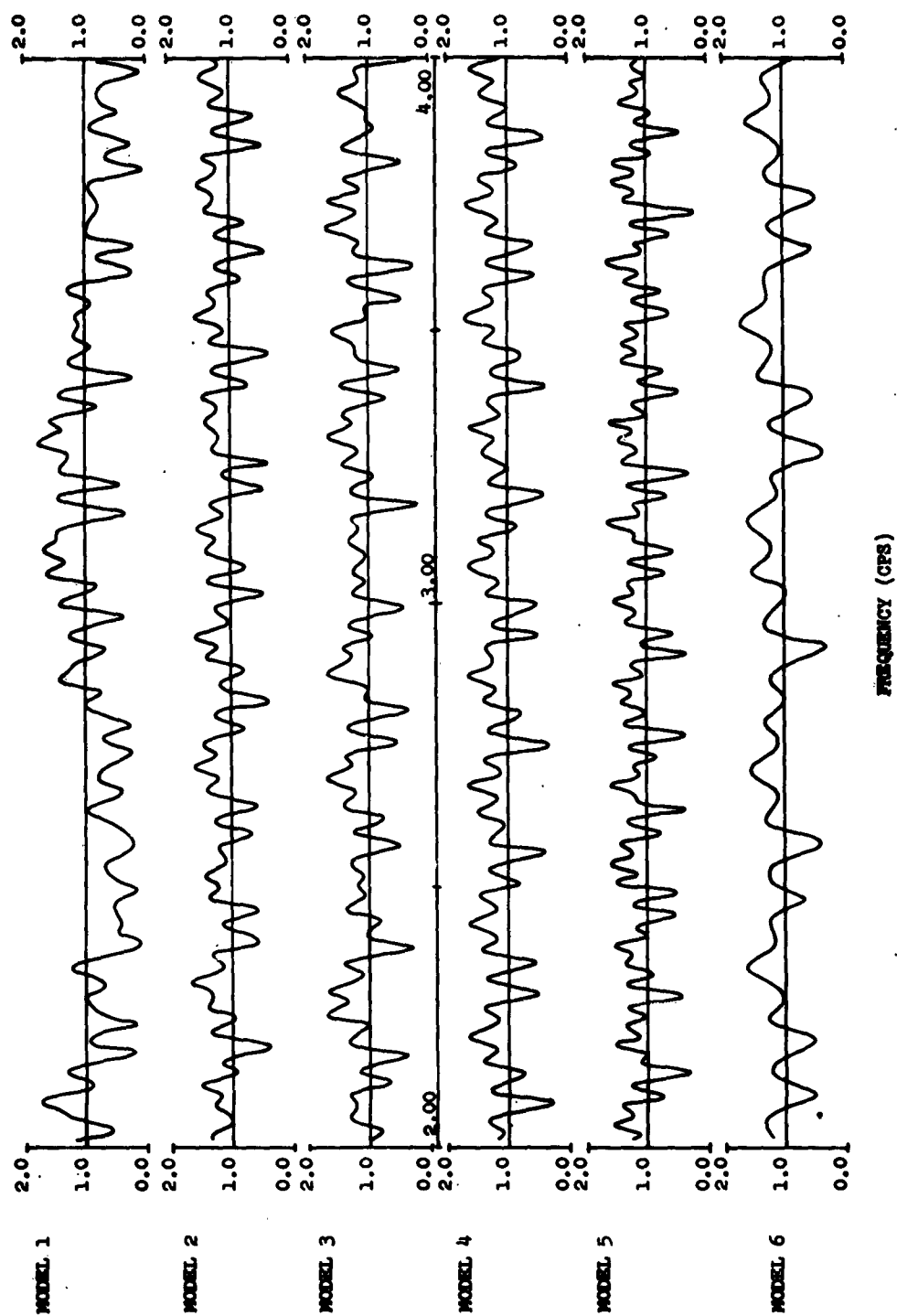
values of TU and TW were used to Fourier synthesize the surface motion
due to an input pulse. Since the values of TU and TW were known in the
range from .02 to 10.0 cps, an incident wave of the form

$$\text{INCIDENT WAVE} = 1/\pi \int_{.04\pi}^{20\pi} \cos pt \, dp$$

was chosen. Upon evaluating the integral we have,

$$\text{INCIDENT WAVE} = (1/\pi) \, ((\sin 20 \pi t)/t - (\sin .04 \pi t)/t).$$

At t=0, this function has the value of 19.96 and on either side of t=0
it appears as a rapidly decaying sine wave. Because the first term is
much larger than the second(by a factor of 500), for small values of t,
the predominant features of this input wave very closely resemble
$(1/\pi)(\sin 20 \pi t)/t$. The value of this function does not exceed 12
per cent of the peak value for $t > .13$ seconds. We combined this
input wave form with TU and TW according to equations (9) in section 2.
These integrals were evaluated for both $u_o(t)$ and $w_o(t)$ for all of the
models at all of the angles. The time range which was chosen for each
angle-model combination was centered on the time of the first arrival
of the surface motion as predicted from independent ray theory calcula-
tions which we carried out. We chose the time interval of the synthesis
at .02 seconds. The synthesized ground motions for the angles of 25,
33, 41 and 49 degrees are shown in Figures 22-29.

In addition to computing the ground motion from TU and TW, we
used the standard ray theory as given in Chapters Two and Three of

FIGURE 22. THE HORIZONTAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE
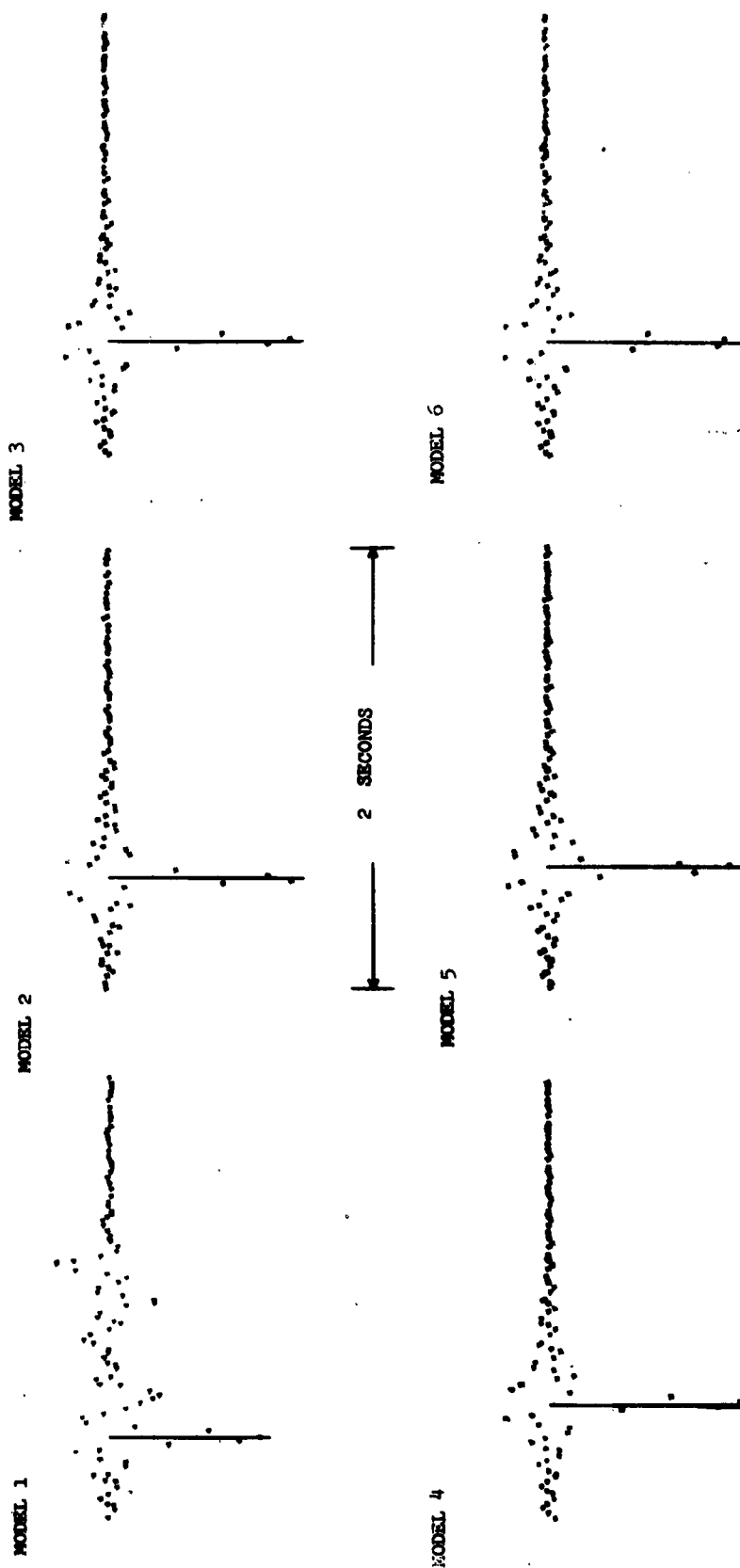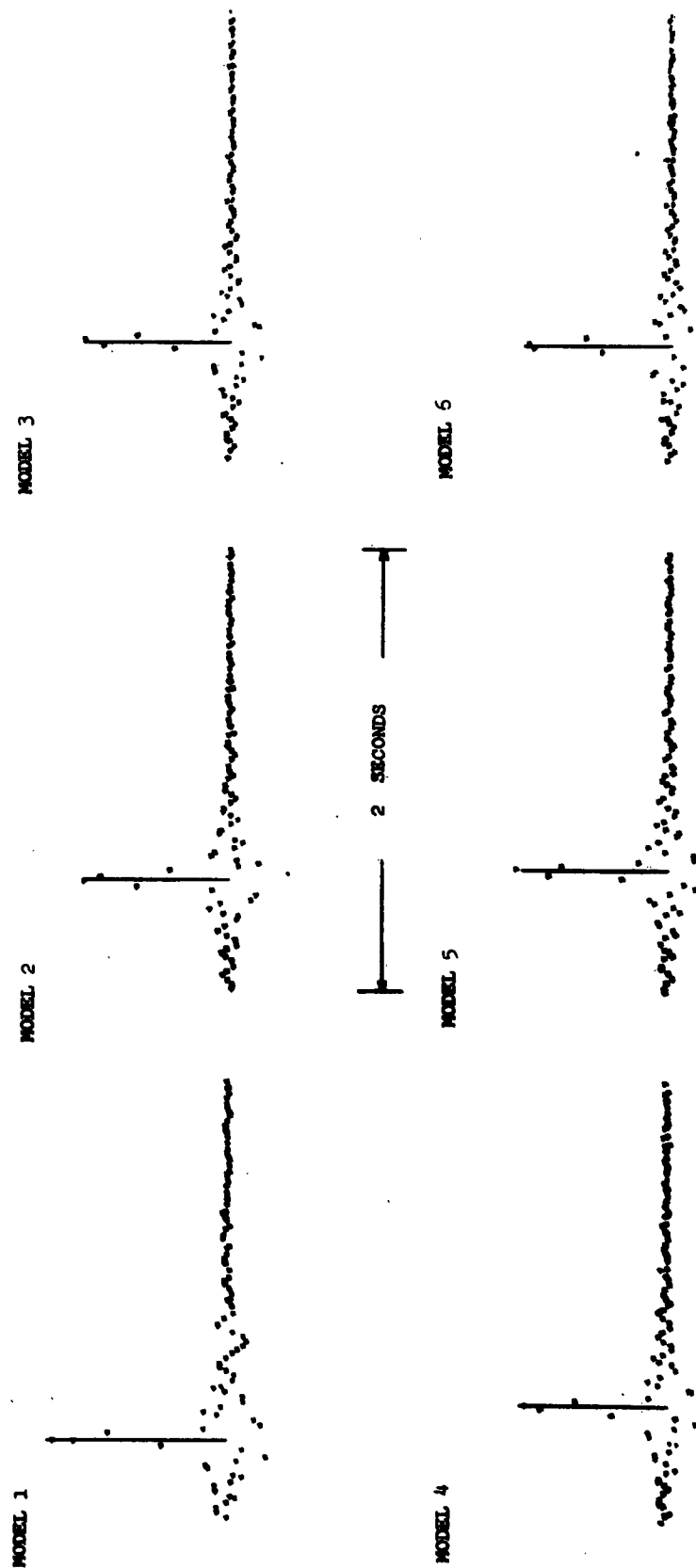MOTION SYNTHESIZED FROM TU FOR AN ANGLE OF INCIDENCE OF 25 DEGREES

FIGURE 23. THE VERTICAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE MOTION
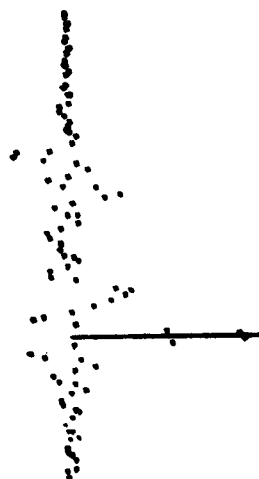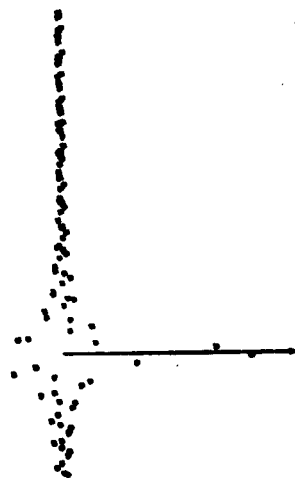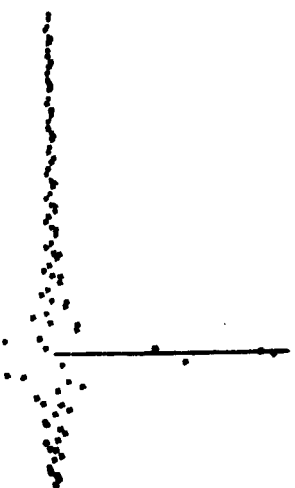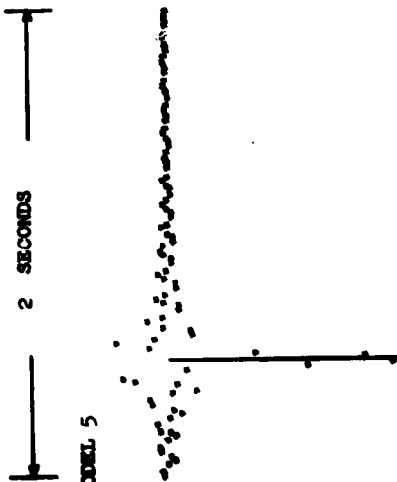SYNTHESIZED FROM TW FOR AN ANGLE OF INCIDENCE OF 25 DEGREES
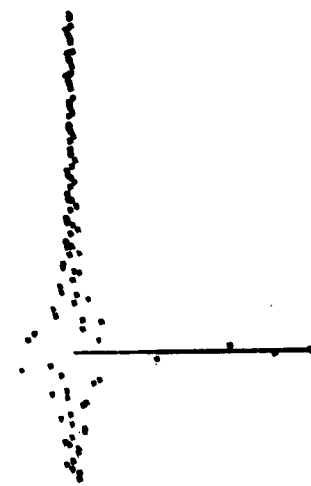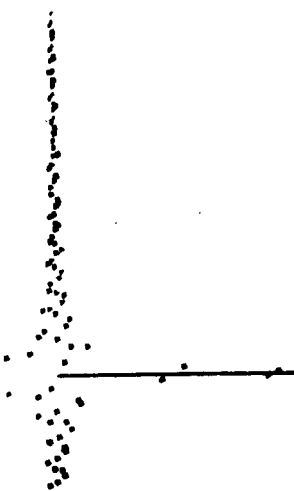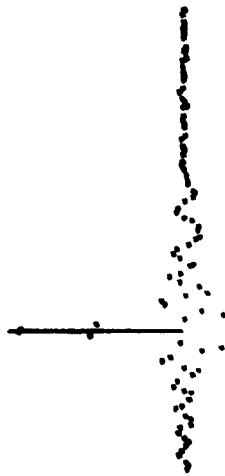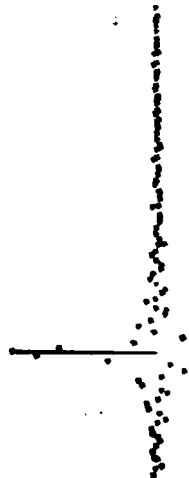
MODEL 1   MODEL 2   MODEL 3

MODEL 4   MODEL 5   MODEL 6

2 SECONDS
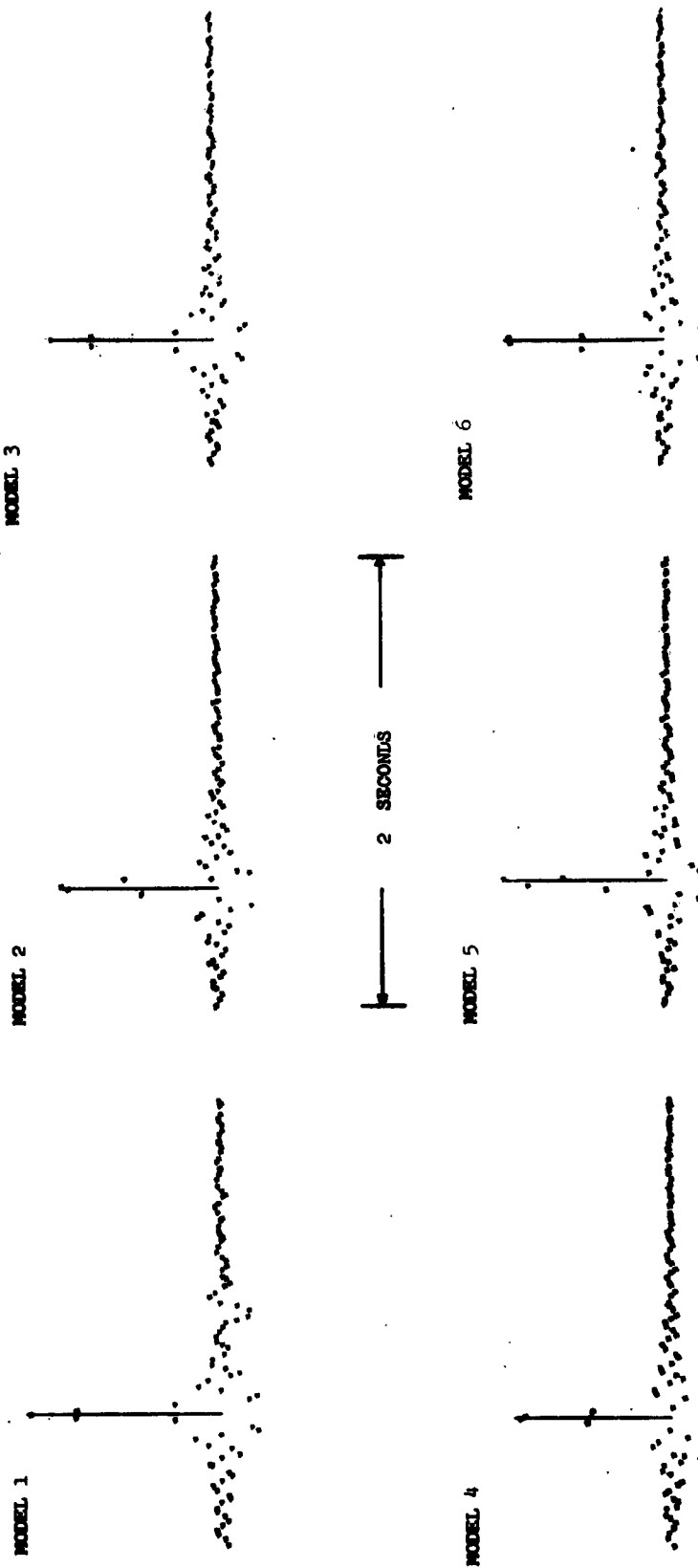
FIGURE 24. THE HORIZONTAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE
MOTION SYNTHESIZED FROM TU FOR AN ANGLE OF INCIDENCE OF 33 DEGREES

FIGURE 25. THE VERTICAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE MOTION SYNTHESIZED FROM TW FOR AN ANGLE OF INCIDENCE OF 33 DEGREES

MODEL 1

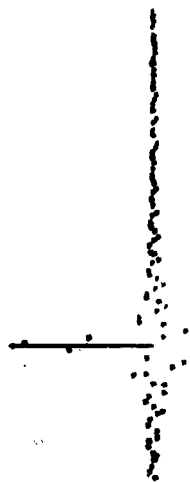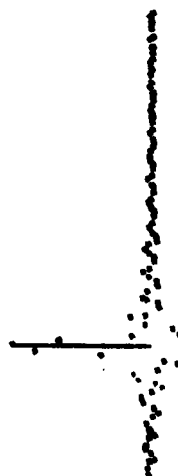MODEL 2

MODEL 3

MODEL 4

MODEL 5

MODEL 6

2 SECONDS

FIGURE 26. THE HORIZONTAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE
MOTION SYNTHESIZED FROM TU FOR AN ANGLE OF INCIDENCE OF 41 DEGREES

MODEL 1

MODEL 2

MODEL 3

2 SECONDS

MODEL 4

MODEL 5

MODEL 6

FIGURE 27. THE VERTICAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE MOTION
SYNTHESIZED FROM TW FOR AN ANGLE OF INCIDENCE OF 41 DEGREES
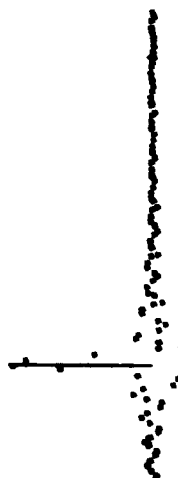
MODEL 3

MODEL 5

MODEL 2

2 SECONDS
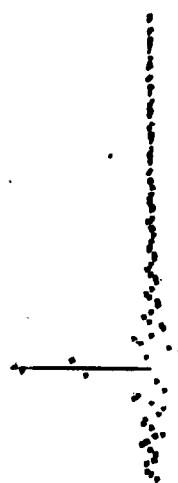
MODEL 5

MODEL 1

MODEL 4

FIGURE 28. THE HORIZONTAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE
MOTION SYNTHESIZED FROM TU FOR AN ANGLE OF INCIDENCE OF 49 DEGREES

MODEL 1

MODEL 2

MODEL 3

MODEL 4
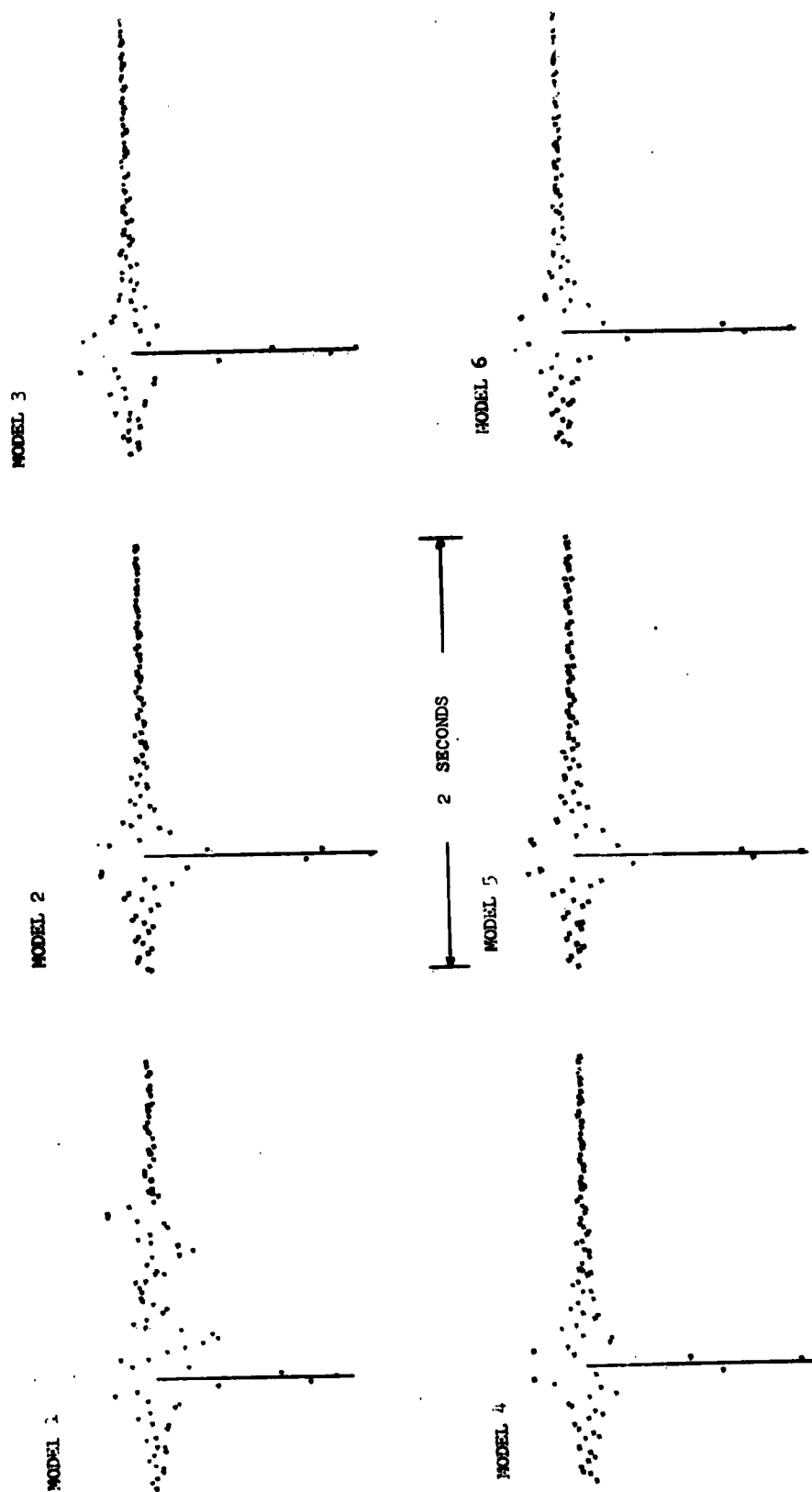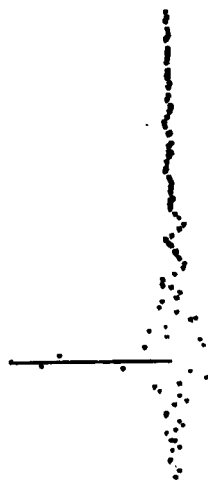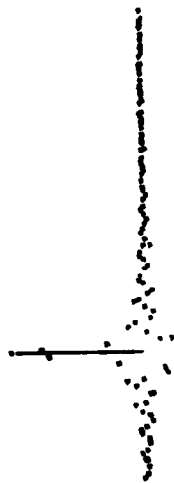
MODEL 5

MODEL 6

2 SECONDS

FIGURE 29. THE VERTICAL COMPONENT OF THE FIRST ARRIVAL OF THE SURFACE MOTION SYNTHESIZED FROM TW FOR AN ANGLE OF INCIDENCE OF 49 DEGREES
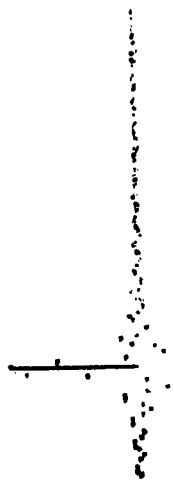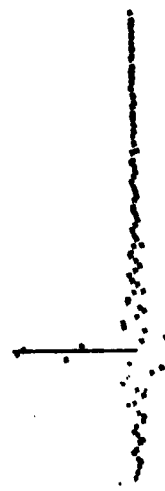
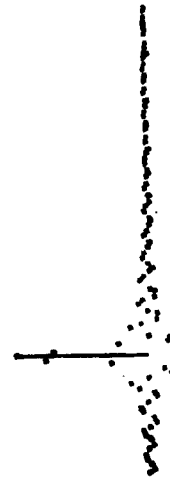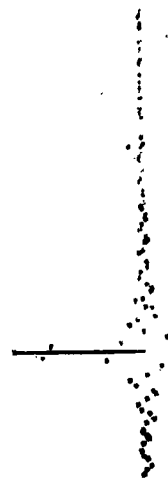Elastic Waves in Layered Media (Ewing, Jardetzky, and Press, 1957) to
compute the arrival time and amplitude of the first arrival. In the
amplitude calculations we took into account the free surface. In doing
so, we assumed that Poisson's ratio was 0.25 in the upper layer for com-
putational simplicity. In these models, the maximum deviation from this
value is .015 or 6 per cent. The relative times and amplitudes of the
ray theory arrivals are shown in the figures by a solid line. We see
that the agreement between the synthesized motion and the amplitudes of
the first arrival based on ray theory is very good. The times agree to
within .007 seconds and the amplitudes to within 6 per cent. The author
considers this agreement to be a substantial check on this work. This
is especially important since most of the calculations cannot be checked
directly.

In considering the synthesized motion we see that the motion in
models 2-6 resembles the time history of the source very closely, while
the motion after the peak arrival in model 1 is quite different. We
would expect this because of the presence of the thin surface layer in
model 1 which is not present in the other models. The arrival times of
these later oscillations agree with those predicted for P to S conver-
sions at the uppermost layer and with multiply reflected P and S waves
within this thin layer. In addition to the work shown in these figures,
we carried out more extensive synthesis at a much coarser time interval,
and we found that the synthesized record showed energy arriving at the
arrival times predicted by ray theory for multiply reflected and con-
verted waves for all of the models. This was taken as a further check
on the validity of these calculations.

4. Summary and Conclusions

In the preceding sections, we have outlined the use of the Haskell-Thomson matrix method to determine the effect of the crust on steady-state dilatational waves. We showed how we could combine a knowledge of the transmission effect of the crust with the recorded surface motion to determine the motion at the base of the crust. Or alternatively, we showed how a knowledge of the surface motion at a single station might be used to determine the crustal structure beneath the station.

Since little is known about the frequency variation of the transmission effect of the crust, we calculated the transmission coefficients for six representative crustal models for nine angles of incidence in the range from 21 degrees to 53 degrees. The frequency variation was considered in the range from .02 to 10.0 cps. Once we determined these coefficients we used them to synthesize the surface motion which would arise from a pulse of the form of $(1/\pi)(\sin 20\pi t)/t$, and thus obtained real time results which could be checked by comparison with the first arrivals predicted by ray theory. From these computations, we can make the following conclusions:

1) Variations in the total thickness of the crust as small as one kilometer cause a noticeable shift in the positions of the peaks and troughs of the transmission coefficients. The thicker the crust, the more rapid is the frequency variation.

2) For crustal models having the same total thickness, the main difference in the frequency variation of the transmission coefficients is in the character of the oscillations. At low frequencies (less than

0.2 cps) this is difficult to determine.

3) The variation in the horizontal transmission coefficient is more irregular than that of the vertical coefficient in the range of angles of incidence studied. This is due to the importance of converted SV waves in this range.

4) The effect of thin, low velocity surface layers is quite pronounced in the transmission coefficients. This effect appears as a large, "long period" oscillation in the frequency variation of the transmission coefficients.

5) A low velocity layer at depth is very difficult to determine, especially from low frequency measurements. However, the difference in shift caused by this layer is observable, and if the transmission coefficients are known over a sufficient range (say .02 to 3.0 cps), it may be possible to observe the presence of the low velocity layer.

6) In general, it is necessary to know the transmission coefficients at intervals of about .015 cps for crustal models of about 38 kilometers thickness in order to observe all of the significant features. For thicker crusts, it will be necessary to know the values at even smaller intervals. Thinner crustal models could be investigated at coarser intervals.

7) The method of determining crustal structure through the use of these transmission coefficients holds considerable promise when the motion after the P arrival shows complicated oscillations which arise from the combination of the input wave form and the crustal interference.

8) When comparing the records obtained from instruments with very different frequency response curves it is important to realize that the sections of the spectrum of the input to which the two instruments are sensitive may be very different. Thus it is possible in extreme cases that short period instruments show substantially larger displacements than long period instruments at the same sites. Further, even the same instruments at different sites at the same distance from the point of generation may show different amplitudes due to the crustal effect. This effect must be taken into account in such investigations as source mechanism studies and magnitude determinations.

9) In those studies in which frequency dependence is important, or in which the observed motion is so complicated that individual events cannot be identified, this method is superior to ray theory in that its application is more direct. However, for models of the type studied here, the two methods are equivalent in that, given the source form and the crustal model, it is possible to predict the surface motion from either one.

Acknowledgment:

APPENDIX I

IBM 1620 COMPUTER PROGRAM FOR TU AND TW

```
C       THIS PROGRAM IS DESIGNED TO COMPUTE THE TRANSFER FUNCTIONS
C       FOR INCIDENT DILATATIONAL WAVES ONLY.
C       APPARENT SURFACE VELOCITY MUST BE GREATER THAN P VELOCITY
C       IN THE TOP LAYER AND THE S VELOCITY IN ANY LAYER
        DIMENSION D(60),A(60),B(60),RHO(60),DGAM(60),DGAM1(60),
       1DRA(60),DRB(60),DH(60)
 1030 PUNCH1
    1 FORMAT(42HP ORIENTED LAYERED SYSTEM RESPONSE 9/25/63)
C       READ IN FOUR LETTER IDENTIFICATION
        READ 1101, T2
 1101 FORMAT(A4)
C       READ IN NO. OF LAYERS AND LAYER CONSTANTS
        READ 2 , NOL
    2 FORMAT(I3)
        PUNCH 2,NOL
C       READ IN LAYER CONSTANTS
        DO 3 I = 1,NOL
        READ 500, D(I) ,A(I),B(I),RHO(I)
    3 PUNCH 500, D(I),A(I),B(I),RHO(I)
  500 FORMAT(4F10.4)
        PUNCH 1620
 1620 FORMAT(2HWV,3X,4HANGI,6X,4HFREQ,5X,7HAPP VEL,5X,1HU,8X,3HPHU,
       18X,1HW,8X,3HPHW,6X,4HODEG)
        N = NOL
        N1 = NOL - 1
        N2 = NOL - 2
C       READ IN INITIAL ANGLE, INCREMENT, FINAL ANGLE IN DEGREES
 7001 READ 4 , ANGIP, DANGI,FANGI
    4 FORMAT(3F10.4)
C       READ IN INITIAL FREQUENCY,INCREMENT, FINAL FREQUENCY
        READ 4, FREQ1,DFREQ,FFREQ
        NOANG = (FANGI - ANGIP)/DANGI + 1.
        NOF = (FFREQ - FREQ1)/DFREQ + 1.
        ANGIP = ANGIP - DANGI
        DO 310 IA = 1,NOANG
        FREQ = FREQ1 - DFREQ
        ANGIP = ANGIP + DANGI
        C = A(NOL)/SINF(ANGIP/57.295780)
C       COMPUTE REUSABLE VARIABLES FOR THE FIRST LAYER
        FCOVA = C/A(1)
        FCOVB = C/B(1)
        FGAM = 2./(FCOVB**2)
        FGAM1 = FGAM -1.
```

```
          FRA = SQRTF(ABSF(FCOVA**2 - 1.))
          FRB = SQRTF(ABSF(FCOVB**2 - 1.))
          FH = RHO(1)*C*C
C         COMPUTE REUSABLE VARIABLES FOR THE REMAINING LAYERS
          DO 1346 M = 2,NOL
          COVA = C/A(M)
          COVB = C/B(M)
          DGAM(M) = 2./(COVB**2 )
          DGAM1(M) = DGAM(M) - 1.
          DRA(M) = SQRTF(ABSF(COVA**2 - 1.)),
          DRB(M) = SQRTF(ABSF(COVB**2 - 1.) )
     1346 DH(M) = RHO(M)*C*C
          DO 310 IFR = 1,NOF
          FREQ = FREQ + DFREQ
          WVNO = 6.2831853*FREQ/C
          P = WVNO*D(1)*FRA
          Q = WVNO*D(1)*FRB
          SINP = SINF(P)
          W = SINP/FRA
          X = FRA*SINP
          COSP = COSF(P)
          SINQ = SINF(Q)
          Y = SINQ/FRB
          Z = FRB*SINQ
          COSQ = COSF(Q)
C         COMPUTE ELEMENTS OF A MATRIX FOR FIRST LAYER
          A11 = FGAM*COSP - FGAM1*COSQ
          A12 = FGAM1*W + FGAM*Z
          A21 = FGAM*X + FGAM1*Y
          A22 = -FGAM1*COSP  + FGAM*COSQ
          A31 = FH*FGAM*FGAM1*(COSP -COSQ)
          A32 = FH*(FGAM1*FGAM1*W + FGAM*FGAM*Z)
          A41 = -FH*(FGAM*FGAM*X + FGAM1*FGAM1*Y)
          A42 = A31
          IF(N2) 1347,1349,1347
C         COMPUTE ELEMENTS OF A MATRIX FOR REMAINING LAYERS
     1347 DO 1345 M = 2,N1
          GAM = DGAM(M)
          GAMM1 = DGAM1(M)
          RA = DRA(M)
          RB = DRB(M)
          H = DH(M)
          P = WVNO*D(M)*RA
          Q= WVNO*D(M)*RB
          IF(C-A(M))121,122,123
      123 SINP = SINF(P)
          W = SINP/RA
          X = RA*SINP
          COSP = COSF(P)
          GO TO 124
      122 W = WVNO*D(M)
```

```
      X = 0.
      COSP = 1.
      GO TO 124
  121 EXPP = EXPF(P)
      EXPM = EXPF(-P)
      W = (EXPP-EXPM)/(2.*RA)
      X= RA*(EXPM-EXPP)/2.
      COSP = (EXPP+EXPM)/2.
  124 SINQ = SINF(Q)
      Y = SINQ/RB
      Z = RB*SINQ
      COSQ = COSF(Q)
      B11 = GAM*COSP - GAMM1*COSQ
      B12 = GAMM1*W + GAM*Z
      B13 = -(COSP-COSQ)/H
      B14 = (W + Z)/H
      B21 =   GAM*X + GAMM1*Y
      B22 = -GAMM1*COSP + GAM*COSQ
      B23 = -( X+Y)/H
      B24 = B13
      B31 = H*GAM*GAMM1*(COSP-COSQ)
      B32 = H*(GAMM1*GAMM1*W + GAM*GAM*Z)
      B33 = B22
      B34 = B12
      B41 = -H*(GAM*GAM*X + GAMM1*GAMM1*Y)
      B42 = B31
      B43 = B21
      B44 = B11
C     MULTIPLY MATRICES
      EA11 = B11*A11 + B12*A21 + B13*A31 + B14*A41
      EA12 = B11*A12 + B12*A22 + B13*A32 + B14*A42
      EA21 = B21*A11 + B22*A21 +B23*A31 + B24*A41
      EA22 = B21*A12 + B22*A22 + B23*A32 + B24*A42
      EA31 = B31*A11 + B32*A21 + B33*A31 + B34*A41
      EA32 = B31*A12 + B32*A22 +B33*A32 + B34*A42
      EA41 = B41*A11 + B42*A21 + B43*A31 + B44*A41
      EA42 = B41*A12 + B42*A22 + B43*A32 + B44*A42
      A11 = EA11
      A12 = EA12
      A21 = EA21
      A22 = EA22
      A31 = EA31
      A32 = EA32
      A41 = EA41
 1345 A42 = EA42
 1349 A21 = -A21
      A41 = -A41
      GAM = DGAM(N)
      GAMM1 = DGAM1(N)
      RA = DRA(N)
```

```
      RB = DRB(N)
      H = DH(N)
C     COMPUTE ELEMENTS OF E INVERSE FOR THE LAST LAYER
      B11 =  - 2.*(B(N)*B(N)) /(A(N)*A(N))
      B13 = 1./(RHO(N)*A(N)*A(N))
      B22 = C*C*GAMM1/(A(N)*A(N)*RA)
      B24 = B13/RA
      B31 = GAMM1/(GAM*RB)
      B33 = -1./(H*GAM*RB)
      B42 = 1.
      B44 = 1./(H*GAM)
      EA11 = B11*A11 + B13*A31
      EA12 = B11*A12 + B13*A32
      EA21 = B22*A21 + B24*A41
      EA22 = B22*A22 + B24*A42
      EA31 = B31*A11 +B33*A31
      EA32 = B31*A12 + B33*A32
      EA41 = B42*A21 + B44*A41
      EA42 = B42*A22 + B44*A42
      OMEG = FREQ*6.2831853
      DR = EA21*EA32 - EA11*EA42 - EA12*EA41 + EA22*EA31
      DI = EA11*EA32+EA21*EA42-EA12*EA31-EA22*EA41
      DENSQ = DR*DR + DI*DI
      UPNR = EA32*DI-EA42*DR
      UPNI = EA32*DR + EA42*DI
      UPDC = (2./DENSQ)*SQRTF(UPNR*UPNR + UPNI*UPNI)
C     UDTP = AMP OF TU,PHUPD = PHASE, WDTP = AMP OF TW, PHWPD = PHASE
      PHUPD = ATANF(UPNI/UPNR)
      PHUPD   = CORANG(PHUPD ,UPNI,UPNR)
      WPNI = EA41*DR +EA31*DI
      WPNR = -EA31*DR + EA41*DI
      WPDC = (2./DENSQ)*SQRTF(WPNR*WPNR+WPNI*WPNI)
      PHWPD = ATANF(WPNI/WPNR)
      PHWPD   = CORANG(PHWPD ,WPNI,WPNR)
      UDTP = COVA*UPDC
      WDTP = COVA*WPDC
  303 FORMAT(A4,F8.3 ,F10.5,F10.3,F10.4,F10.5,F10.4,F10.5,F8.3)
  310 PUNCH 303,T2,ANGIP,FREQ,C,UDTP,PHUPD ,WDTP,PHWPD ,OMEG
C     CONTROL CARD TO CONTROL RECYCLE
 1020 READ 7000, CNTRL
 7000 FORMAT(F10.2)
      IF(CNTRL) 7001,1030,1021
 1021 PRINT 990
  990 FORMAT(11HEND OF CASE)
      Y = LINK(EA31)
      END
C     CORANG IS A SUBROUTINE TO DETERMINE THE RIGHT QUADRANT
C     FOR THE ANGLE DETERMINED BY ATANF
      FUNCTION CORANG(FUA,FUI,FUR)
      IF(FUA) 1,1,4
```

```
1 IF(FUI) 2,2,3
2 ANGMUL = 0.
  GO TO 7
3 ANGMUL = 1.
  GO TO 7
4 IF(FUI) 5,5,6
5 ANGMUL =-1.
  GO TO 7
6 ANGMUL = 0.
7 CORANG = FUA + ANGMUL*3.14159265
  RETURN
  END
```

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Unclassified Report

The effect of crustal model on the variation of surface motion with angle of incidence is examined by an application of the matrix formulation of N. A. Haskell. Six crustal models, including such features as low velocity surface layers, low velocity layers at depth, and relatively thick and relatively thin total thickness are examined. Transmission coefficients are computed for frequencies ranging from .02 cps to 10.0 cps, and for angle of incidence ranging from 21 degrees to 53 degrees. The surface motion due to an incident wave of the form $(1/\pi)(\sin 20\pi t)/t$ is computed.

1. Seismic Waves
2. Transmission coefficients
3. Layered Media
4. Seismology
I. Contract AF 19(604)-7399
II. St. Louis University, St. Louis, Mo.
III. Hannon
IV. In DDC collection
Project 8652, Task 865201